YEAR 9 – REASONING WITH ALGEBRA...

Straight Line Graphs

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Compare gradients
- Compare intercepts
- Understand and use y= mx + c
- Find the equation of a line from a graph
- Interpret gradient and intercepts of real-life graphs

Keywords

Gradient: the steepness of a line

Intercept: where two lines cross. The y-intercept: where the line meets the y-axis.

Parallel: two lines that never meet with the same gradient.

Co-ordinate: a set of values that show an exact position on a graph.

Linear: linear graphs (straight line) – linear common difference by addition/ subtraction

Asymptote: a straight line that a graph will never meet.

Reciprocal: a pair of numbers that multiply together to give 1.

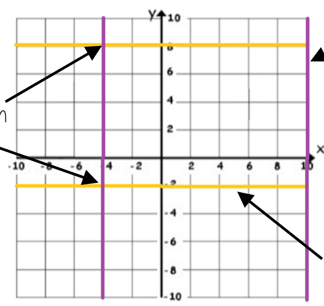
Perpendicular: two lines that meet at a right angle.

Lines parallel to the axes (R)

'a' can be ANY positive or negative value including 0

All the points on this line have a x coordinate of 10

Intersection points



Lines parallel to the **y axis** take the form **x = a** and are **vertical**

Lines parallel to the **x axis** take the form **y = a** and are **horizontal**

All the points on this line have a y coordinate of -2

eg (3, -2) (7, -2) (-2, -2) all lay on this line because the y coordinate is -2

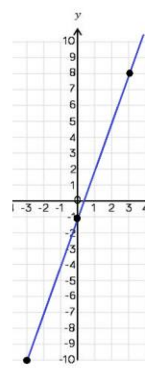
Plotting y = mx + c graphs (R)

$y = 3x - 1$ → 3 x the x coordinate then – 1

x	−3	0	3
y	-10	-1	8

Draw a table to display this information

This represents a coordinate pair (-3, -10)



You only need two points to form a straight line

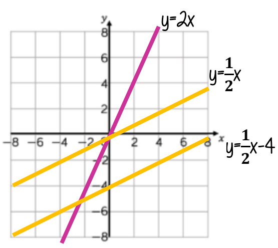
Plotting more points helps you decide if your calculations are correct (if they do make a straight line)

Remember to join the points to make a line

Compare Gradients

$y = mx + c$

The **coefficient** of x (the number in front of x) tells us the gradient of the line



The **greater** the gradient – the **steeper** the line

Parallel lines have the **same** gradient

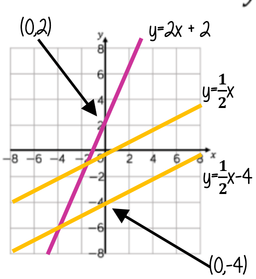
Positive gradients

Negative gradients

Compare Intercepts

$y = mx + c$

The value of **c** is the point at which the line crosses the y-axis. **Y intercept**



The coordinate of a y intercept will always be (0,c)

Lines with the **same** y-intercept cross in the **same** place

y = mx + c

The **coefficient** of x (the number in front of x) tells us the gradient of the line

$y = mx + c$

The value of **c** is the point at which the line crosses the y-axis. **Y intercept**

y and x are **coordinates**

The equation of a line can be rearranged. Eg:

y = c + mx

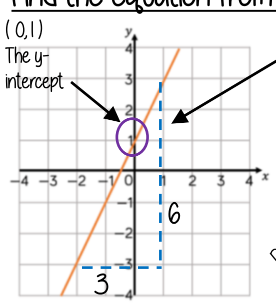
c = y – mx

Identify which coefficient you are identifying or comparing

Find the equation from a graph

(0,1) The y-intercept

The Gradient $\frac{6}{3} = 2$



$y = 2x + 1$

The direction of the line indicates a positive gradient

Positive gradients

Negative gradients

Real life graphs

A plumber charges a £25 callout fee, and then £12.50 for every hour. Complete the table of values to show the cost of hiring the plumber.

Time (h)	0	1	2	3	8
Cost (£)	£25				£125

The y-intercept shows the minimum charge. The gradient represents the price per mile

In real life graphs like this values will always be positive because they measure distances or objects which cannot be negative.

Direct Proportion graphs To represent direct proportion the graph must start at the origin.

When you have 0 pens this has 0 cost. The gradient shows the price per pen

A box of pens costs £2.30

Complete the table of values to show the cost of buying boxes of pens.

Boxes	0	1	2	3	8
Cost (£)		£2.30			

YEAR 9 – REASONING WITH ALGEBRA…
Forming and Solving Equations

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Solve inequalities with negative numbers
- Solve equations with unknowns on both sides
- Solve inequalities with unknowns on both sides
- Substitute into formulae and equations
- Rearrange formulae

Keywords

Inequality: an inequality compares who values showing if one is greater than, less than or equal to another

Variable: a quantity that may change within the context of the problem

Rearrange: Change the order

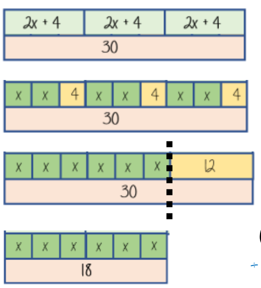
Inverse operation: the operation that reverses the action

Substitute: replace a variable with a numerical value

Solve: find a numerical value that satisfies an equation

Solve equations with brackets

R



$3(2x + 4) = 30$

Expand the brackets

$6x + 12 = 30$

$-12 \quad -12$

$6x = 18$

$\div 6 \quad \div 6$

$\underline{x = 3}$

Form and solve inequalities

R

Two more than treble my number is greater than 11

Find the possible range of values

$3x + 2 > 11$

Solve

x ← −3 ← −2 ← 11

$x > 3$

Inequalities with negatives

Method 1 Make x positive first

$2 - 3x > 17$

$+3x \quad +3x$

$2 > 17 + 3x$

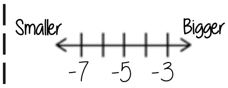
$-17 \quad -17$

$-15 > 3x$

$\div 3 \quad \div 3$

$\underline{-5 > x}$

x is true for any value smaller than -5



✓ CHECK IT!
$2 - 3(-6) = 20$
TRUE/ CORRECT

Method 2 Keep the negative x

$2 - 3x > 17$

$-2 \quad -2$

$-3x > 15$

$\div -3 \quad \div -3$

$\underline{x > -5}$

x is true for any value bigger than -5

This cannot be true…

$\underline{x < -5}$

When you multiply or divide x by a negative you need to reverse the inequality

Equations with unknown on both sides

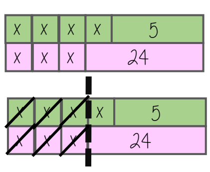
$4x + 5 = 3x + 24$

$-3x \quad -3x$

$x + 5 = 24$

$-5 \quad -5$

$\underline{x = 19}$



Inequalities with unknown on both sides

Solving inequalities has the same method as equations

$5(x + 4) < 3(x + 2)$

$5x + 20 < 3x + 6$

$2x + 20 < 6$

$2x < -14$

$\underline{x < -7}$

Check it!

$5(-8 + 4) < 3(-8 + 2)$

$5(-4) < 3(-6)$

$-20 < -18$

✓ -20 IS smaller than -18

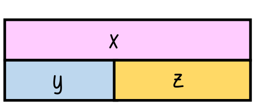
Formulae and Equations

Formulae – all expressed in symbols

Substitute in values →

Equations – include numbers and can be solved

Rearranging Formulae (one step)



$x = y + z$

Rearrange to make y the subject.

$\underline{y = x - z}$

y → +z → x

y ← −z ← x

Using inverse operations or fact families will guide you through rearranging formulae

Rearranging can also be checked by substitution.

Language of rearranging…

Make XXX the subject

Change the subject

Rearrange

Rearranging Formulae (two step)

In an equation (find x)

$4x - 3 = 9$

$+3 \quad +3$

$4x = 12$

$\div 4 \quad \div 4$

$\underline{x = 3}$

In a formula (make x the subject)

$xy - s = a$

$+s \quad +s$

$xy = a + s$

$\div y \quad \div y$

$x = \dfrac{a + s}{y}$

The steps are the same for solving and rearranging

Rearranging is often needed when using $y = mx + c$

e.g. Find the gradient of the line $2y - 4x = 9$

Make y the subject first $y = \dfrac{4x + 9}{2}$ Gradient $= \dfrac{4}{2} = 2$

YEAR 9 – REASONING WITH ALGEBRA...
Testing conjectures

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Use factors, multiples and primes
- Reason True or False
- Reason Always, sometimes never true
- Show that reasoning
- Make conjectures about number
- Expand binomials
- Make conjectures with algebra
- Explore the 100 grid

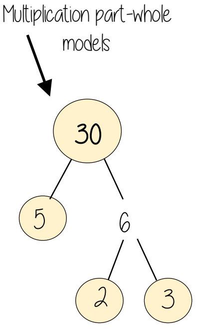
Keywords

Multiples: found by multiplying any number by positive integers
Factor: integers that multiply together to get another number.
Prime: an integer with only 2 factors.
HCF: highest common factor (biggest factor two or more numbers share)
LCM: lowest common multiple (the first time the times table of two or more numbers match)
Verify: the process of making sure a solution is correct
Proof: logical mathematical arguments used to show the truth of a statement
Binomial: a polynomial with two terms
Quadratic: a polynomial with four terms (often simplified to three terms)

Factors, Multiples and Primes

R

Multiplication part-whole models



All three prime factor trees represent the same decomposition

HCF – Highest common factor

HCF of 18 and 30

18	1, 2, 3, 6, 9, 18
30	1, 2, 3, 5, 6, 10, 15, 30

Common factors are factors two or more numbers share

LCM – Lowest common multiple

LCM of 9 and 12

9	9, 18, 27, 36, 45, 54
12	12, 24, 36, 48, 60

Common multiples are multiples two or more numbers share

True or False?

Conjecture

A pattern that is noticed for many cases

1, 2, 4.... The numbers in the sequence are doubling each time.

Counterexamples

This sequence isn't doubling it is adding 2 each time

Only one counterexample is needed to disprove a conjecture

Always, Sometimes, Never true.

Always	Every value always supports the statement
Sometimes	Examples show the statement being true and counter examples to show when it is false.
Never	No example supports the statement

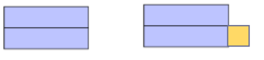
Examples to try
- 0 and 1
- Fractions
- Negative numbers

Show that

Numerical verification	Show the stages to a solution with numerical values
Algebraic verification	Show algebraic properties of the solution. You may want to use pictorial images to support this
Proof	Simple proofs using algebra

Compare the left hand side of an equation with the right hand side – are they the same or different?

Conjectures



Even $(2n)$ Multiple of 2

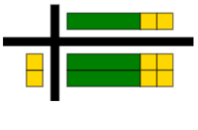
Odd $(2n+1)$ One more than any even

Use numerical verification first

Use pictorial verification – the representations of numbers of odd and even

Expanding binomials

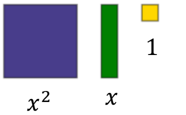
$2(x+2) \equiv 2x+4$



Algebra tiles can represent a binomial expansion

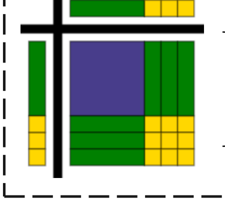
Has two terms

Algebra tiles



Positive values

$(x+3)(x+3) \equiv x^2+6x+9$



This is a quadratic. It has four terms which simplified to three terms

The order of the binomial has no impact on the outcome. e.g. $(x+3)(3+x)$

Exploring the 100 square

In terms of 'n' is used to make generalisations about relationships between numbers

Positions of numbers in relation to n form expressions.

Eg one space to the right of n
$n + 1$

Eg One row below n
$n + 10$

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

The size of the grid for generalisation changes the relationship statements

YEAR 9 – CONSTRUCTING IN 2D/3D…
3D Shapes

@whisto_maths

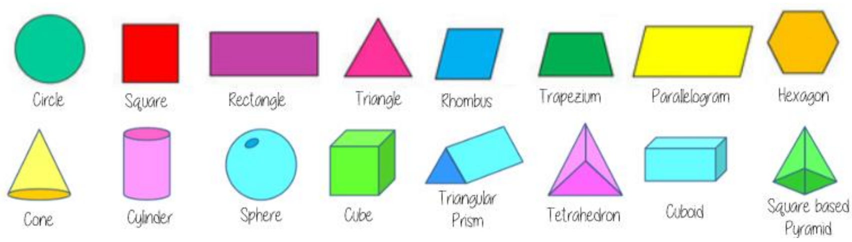
What do I need to be able to do?

By the end of this unit you should be able to:
- Name 2D & 3D shapes
- Recognise Prisms
- Sketch and recognise nets
- Draw plans and elevations
- Find areas of 2D shapes
- Find Surface area for cubes, cuboids, triangular prisms and cylinders
- Find the volume of 3D shapes

Keywords

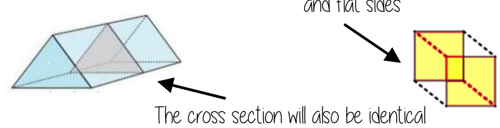
2D: two dimensions to the shape e.g. length and width
3D: three dimensions to the shape e.g. length, width and height
Vertex: a point where two or more line segments meet
Edge: a line on the boundary joining two vertex
Face: a flat surface on a solid object
Cross-section: a view inside a solid shape made by cutting through it
Plan: a drawing of something when drawn from above (sometimes birds eye view)
Perspective: a way to give illustration of a 3D shape when drawn on a flat surface.

Name 2D & 3D shapes



Recognise prisms

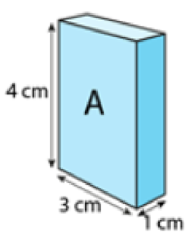
A solid object with two identical ends and flat sides

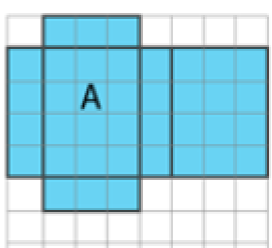


The cross section will also be identical to the end faces.

A cylinder although with very similar properties does not have flat faces so is not categorised as a prism

Nets of cuboids

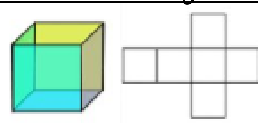




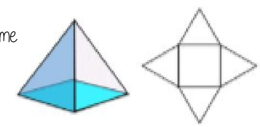
1cm grids help to draw accurately

Visualise the folding of the net. Will it make the cuboid with all sides touching

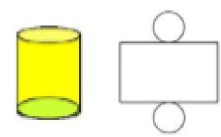
Sketch and recognise nets



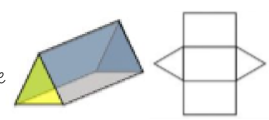
Do they have the same number of faces?



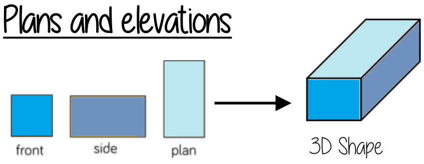
Where do the edges join?



Are the shapes of the faces correct?



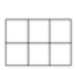
Plans and elevations



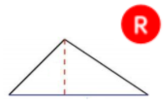
The direction you are considering the shape from determines the front and side views

Area of 2D shapes

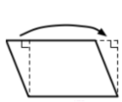
Rectangle
Base x Height



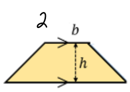
Triangle
½ x Base x Perpendicular height



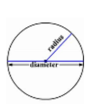
Parallelogram/ Rhombus
Base x Perpendicular height



Area of a trapezium
$\frac{(a+b) \times h}{2}$

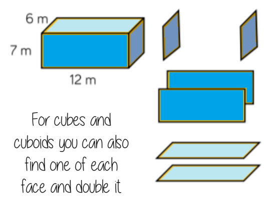


Area of a circle
$\pi \times radius^2$



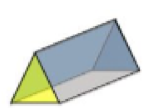
Surface area

Sketching nets first helps you visualise all the sides that will form the overall surface area



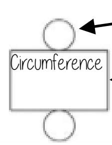
Sides	6 x 7	Sum of all sides is surface area
	6 x 7	
Front and back	12 x 7	
	12 x 7	
Top and Bottom	12 x 6	
	12 x 6	

For cubes and cuboids you can also find one of each face and double it



For other shapes – not all the sides are the same, so calculate the individually

Surface area – cylinders



The area of the circle
$\pi \times radius^2$

The width of this face is the same as the circumference
$\pi \times diameter \times height$

$2 \times \pi \times radius^2 + \pi \times diameter \times height$

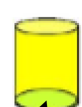
Volumes

Volume is the 3D space it takes up – also known as capacity if using liquids to fill the space

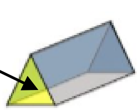
Counting cubes
Some 3D shape volumes can be calculated by counting the number of cubes that fit inside the shape

Cubes/ Cuboids = base x width x height

Remember multiplication is commutative



Cross section



Cross section

Prisms and cylinders
= area cross section x height

Height can also be described as depth

Areas – square units
Volumes – cube units

Areas and volumes can be left in terms of pi π

YEAR 9 – CONSTRUCTING IN 2D/3D…

Constructions & congruency

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What do I need to be able to do?

By the end of this unit you should be able to:

- Draw and measure angles
- Construct scale drawings
- Find locus of distance from points, lines, two lines
- Construct perpendiculars from points, lines, angles
- Identify congruence
- Identify congruent triangles

Keywords

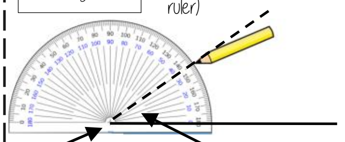
Protractor: piece of equipment used to measure and draw angles
Locus: set of points with a common property
Equidistant: the same distance
Discorectangle: (a stadium) – a rectangle with semi circles at either end
Perpendicular: lines that meet at 90°
Arc: part of a curve
Bisector: a line that divides something into two equal parts
Congruent: the same shape and size

Draw and measure angles

R

Draw a 35° angle

Make a mark at 35° with a pencil
And join to the angle point (use a ruler)



The angle

Make sure the cross is at the end of the line (where you want the angle)

Scale drawings

R

A picture of a car is drawn with a scale of 1:30

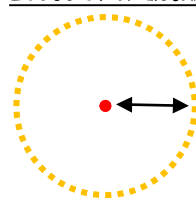
For every 1cm on my image is 30cm in real life

The car image is 10cm

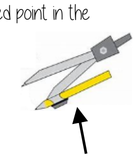
Image	:	Real life
1cm	:	30cm
10cm	:	300cm

× 10 (both sides)

Locus of a distance from a point



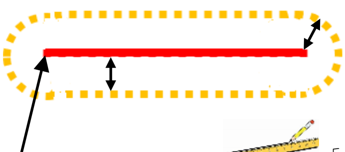
All points are equidistant (the same distance) from the fixed point in the middle.



Equipment needed
The radius is the distance from the fixed point

If the point is in the corner it can only make a quarter circle

Locus of a distance from a straight line

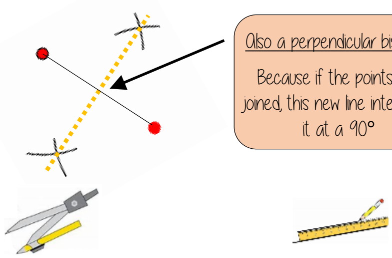


All points are equidistant (the same distance) from line

The ends of the line are fixed points

Equipment needed
The line is straight so a ruler is used for the straight lines parallel to your original line

Locus equidistant from two points



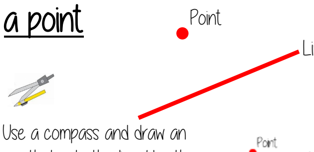
Also a perpendicular bisector

Because if the points are joined, this new line intersects it at a 90°

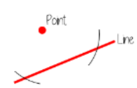
Keep the compass the same size and draw two arcs from each point

Join the intersections with a ruler.
All points on this line are equidistant from both points

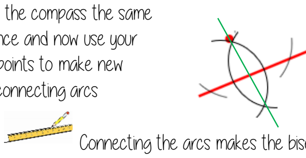
Construct a perpendicular from a point



Use a compass and draw an arc that cuts the line. Use the point to place the compass



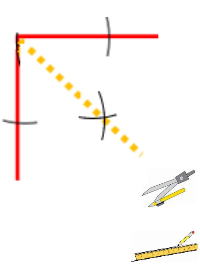
Keep the compass the same distance and now use your new points to make new interconnecting arcs



Connecting the arcs makes the bisector

If P is a point on the line the steps are the same

Locus of a distance from two lines



Also an angle bisector
This cuts the angle in half

From the angle vertex draw two arcs that cut the lines forming the angle

Keep the compass the same size and use the new arcs as centres to draw intersecting arcs in the middle

Join the vertex to the intersection

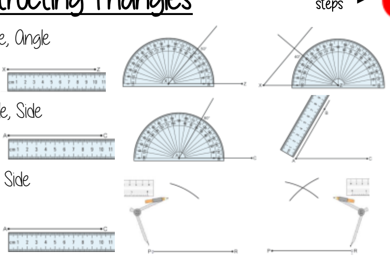
Constructing Triangles

Link to steps → R

Side, Angle, Angle

Side, Angle, Side

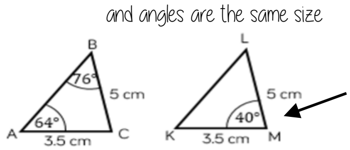
Side, Side, Side



Congruent figures

Congruent figures are identical in size and shape – they can be reflections or rotations of each other

Congruent shapes are identical – all corresponding sides and angles are the same size



$A\hat{C}B = K\hat{M}L$

Because all the angles are the same and AC=KM BC=LM triangles ABC and KLM are **congruent**

Congruent triangles

Side-side-side

All three sides on the triangle are the same size

Angle-side-angle

Two angles and the side connecting them are equal in two triangles

Side-angle-side

Two sides and the angle in-between them are equal in two triangles (it will also mean the third side is the same size on both shapes)

Right angle-hypotenuse-side

The triangles both have a right angle, the hypotenuse and one side are the same

YEAR 9 – REASONING WITH NUMBER...
Numbers

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Identify integers, real and rational numbers
- Work with directed number
- Solve problems with number
- Find HCF/ LCM
- Add/ Subtract fractions
- Multiply/ Divide fractions
- Write numbers in standard form

Keywords

Integer: a whole number that is positive or negative
Rational: a number that can be made by dividing two integers
Irrational: a number that cannot be made by dividing two integers
Inverse operation: the operation that reverses the action
Quotient: the result of a division
Product: the result of a multiplication
Multiples: found by multiplying any number by positive integers
Factor: integers that multiply together to get another number

Integers, real and rational numbers

Rational – root word: ratio

Real numbers: $\frac{2}{3}$ stems from 2:1 ($\frac{2}{3}$ of the whole)

Irrational numbers: $\sqrt{2}$ the solution is a decimal that never ends and does not repeat.

The square root of a negative is not a real number and cannot be found

HCF/LCM (R)

1 is a common factor of all numbers

Common factors are factors two or more numbers share

HCF – Highest common factor

HCF of 18 and 30

18	1, 2, 3, (6), 9, 18
30	1, 2, 3, 5, (6), 10, 15, 30

HCF = 6

LCM – Lowest common multiple

LCM of 9 and 12

9	9, 18, 27, (36), 45, 54
12	12, 24, (36), 48, 60

LCM = 36

The first time their multiples match

Standard form (R)

Any number between 1 and less than 10 → $A \times 10^n$ ← Any integer

$6 \times 10^5 + 8 \times 10^5$

= 600000 + 800000
= 1400000
= 1.4×10^5

$(1.5 \times 10^5) \div (0.3 \times 10^3)$

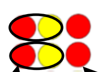
$1.5 \div 0.3 \quad \times 10^5 \div 10^3$
= 5×10^2

Directed number (R)

(red counter) = –1, (yellow counter) = 1

Addition

2 + - 4 = -2

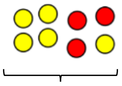


Zero pair (-1 + 1 = 0)

Two "– 1" left = -2

Generalisation: + – = –

Subtraction



= 2

Representation for calculation

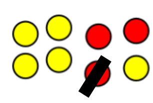
Generalisation: – – = +

"Subtract" – means take away or remove

2 - - 1 = 3

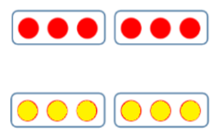
Take away one (red counter)

Start with the representation of 2



= 3

Multiplication



The act of making counters into their negative is turning them over

-2 x -3 = 6

Divisions are the inverse operations

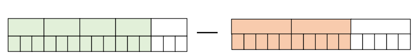
a = 5 b = -4

Brackets around negative substitutions helps remove calculation errors

2a – b = 2 x 5 – (-4) = 10 + 4 = 14

Addition/ Subtraction of fractions (R)

$\frac{4}{5} - \frac{2}{3}$



$\frac{12}{15} - \frac{10}{15} = \frac{2}{15}$

Use equivalent fractions to find a common multiple for both denominators

Multiplication/ Division of fractions (R)

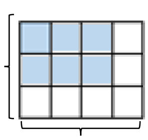
$\frac{3}{4} \times \frac{2}{3}$

Shade in 3 parts; Repeat it on this many rows; This many columns; This many rows

$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$

Parts shaded; Total number of parts in the diagram

Modelled:



3, 4

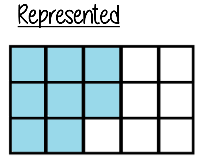
Remember to use reciprocals

$\frac{2}{5} \div \frac{3}{4}$

$\frac{2}{5} \times \frac{4}{3}$

Multiplying by a reciprocal gives the same outcome

Represented



$= \frac{8}{15}$

YEAR 9 – REASONING WITH NUMBER...

Using Percentages

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Use FDP equivalence
- Calculate percentage increase and decrease
- Express percentage change
- Solve reverse percentage problems
- Solve percentage problems (calculator and non calculator problems)

Keywords

Percent: parts per 100 – written using the % symbol

Decimal: a number in our base 10 number system. Numbers to the right of the decimal place are called decimals.

Fraction: a fraction represents how many parts of a whole value you have.

Equivalent: of equal value.

Reduce: to make smaller in value.

Growth: to increase/ to grow.

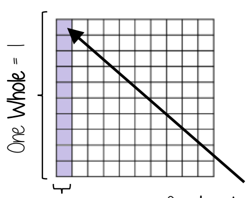
Integer: whole number, can be positive, negative or zero.

Invest: use money with the goal of it increasing in value over time (usually in a bank).

Multiplier: the number you are multiplying by.

Profit: the income take away any expenses/ costs.

FDP Equivalence (R)



Percentage

100% = a whole = 100 hundredths

10 hundredths
10 out of 100
10%

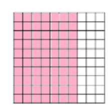
$\frac{10}{100} = \frac{1}{10} = 0.10$

One hundredth (one whole split into 100 equal parts)

ones	tenths	hundredths
•	●	

Converting FDP (R)

$\frac{70}{100}$ → This also means 70 ÷ 100 → 70 out of 100 squares 70 "hundredths" = 7 "tenths" 0.7 →



→ 70 hundredths = 70%

Using a calculator

→ S⇔D Convert to a decimal → × 100 converts to a percentage

Be careful of recurring decimals

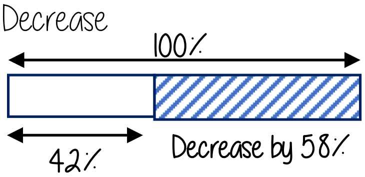
eg $\frac{1}{3}$ = 0.3333333

= $0.\dot{3}$

The dot above the 3

Percentage Increase/ Decrease (R)

Decrease

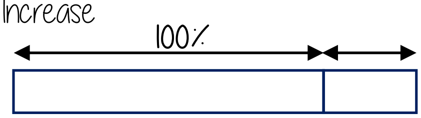


Decrease by 58%

Multiplier Less than 1

1.00 – 0.58 = 0.42

Increase



Increase by 12%

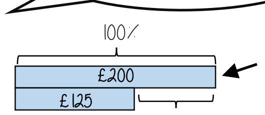
100% + 12% = 112%

1.00 + 0.12 = 1.12

Multiplier More than 1

Percentage change (R)

I bought a phone for £200.
A year later sold it for £125.



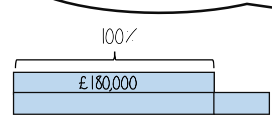
All values of change compare to the ORIGINAL value

Percentage loss

$\frac{75}{200} \times 100 = 37.5\%$

$$\frac{\text{Difference in values}}{\text{Original value}} \times 100$$

I bought a house for £180,000, I later sold it for £216,000.

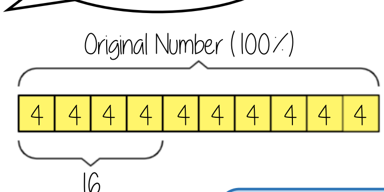


Percentage profit

Money made (profit value) → $\frac{36000}{180000} \times 100 = 20\%$

Reverse Percentages

40% of my number is 16. What am I thinking of?



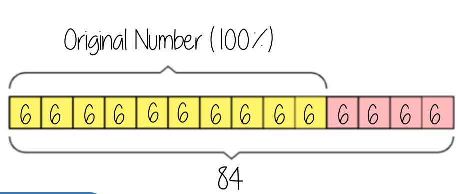
40% = 16

10% – 4

100% = 40

Try to scale down to 10% or 1% and then scale back up to 100%

140% of my number is 84. What is the original number?



140% = 84

10% – 6

100% = 60

YEAR 9 – REASONING WITH NUMBER…
Maths & Money

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:
- Solve problems with bills and bank statements
- Calculate simple interest
- Calculate compound interest
- Calculate wages and taxes
- Solve problems with exchange rates
- Solve unit pricing problems

Keywords

Credit: money being placed into a bank account
Debit: money that leaves a bank account
Balance: the amount of money in a bank account
Expense: a cost/ outgoing
Deposit: an initial payment (often a way of securing an item you will later pay for)
Multiplier: a number you are multiplying by. (Multiplier more than 1 = increasing, less than 1 = decreasing)
Per Annum: each year
Currency: the type of money a country uses.
Unitary: one – the cost of one.

Bills and Bank Statements

Bills – tell you the amount items cost and can show how much money you need to pay

Some can include a total

Look for different units (Is it in pence or pounds)

Menu	Price
Milk	89p
Tea	£1.50

Bank Statements

Bank statement can have negative balances if the money spent is higher than the money coming into the account

Date	Description	Credit	Debit	Balance
19th Sept	Salary	£1500		£1500
19th Sept	Mortgage		£600	£900
25th Setp	Bday Money	£15		£915

Simple Interest

For each year of investment the interest remains the same

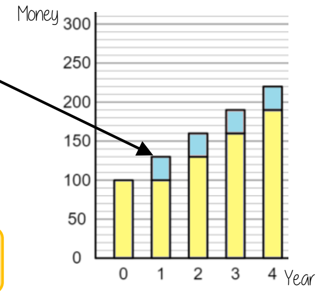
$$\frac{\text{Principal amount} \times \text{Interest Rate} \times \text{Years}}{100}$$

Principal amount is the amount invested in the account.

e.g. Invest £100 at 30% simple interest for 4 years

$$\frac{100 \times 30 \times 4}{100} = £120$$

This account earned **£120** interest.
At the end of year 4 they have **£220**



Compound Interest

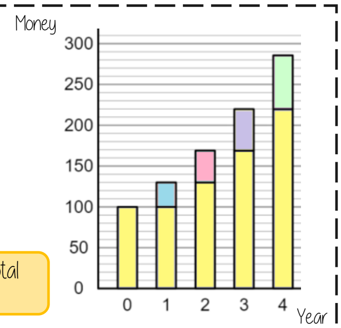
Interest is added to the current value of investment at the end of each year so the next year's interest is greater.

$$\text{Principal amount} \times \text{Multiplier}^{\text{Years}}$$

e.g. Invest £100 at 30% compound interest for 4 years

$$100 \times 1.3^4 = £285.61$$

This account has **£285.61** in total at the end of the 4 years.



Value Added Tax (VAT)

VAT is payable to the government by a business. In the UK VAT is 20% and added to items that are bought.

Essential items such as food do not include VAT.

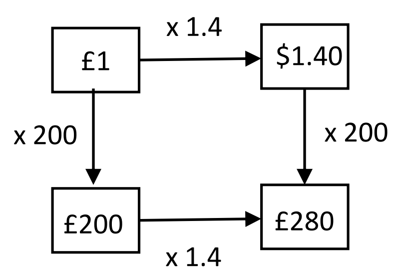
Wages and Taxes

Salaries fall into tax brackets – which means they pay this much each month from their salary.

Taxable Income	Tax Rate
£12 501 to £50 000	20%
£50 001 to £150 000	40%
over £150 000	45%

Over time:
Time and a half – means 1.5 times their hourly rate
Double – 2 times their hourly rate

Exchange Rates



When making estimates it is also useful to use estimates to check if our solution is reasonable.

Use inverse operations to reverse the exchange process

Common Currencies		
United Kingdom	£	Pounds
United States of America	$	Dollars
Europe	€	Euros

Unit Pricing

4 Oranges	5 cupcakes
£1	£1.20

$4 = £1.00$
$\div 2$
$2 = £0.50$
$\div 2$
$1 = £0.25$

$5 = £1.20$
$\div 5$
$1 = £0.20$

Cost per Unit

To calculate unit per cost you divide by the cost.

Cupcakes are the best value as one item has the cheapest value

There is a directly proportional relationship between the cost and number of units.

YEAR 9 – REASONING WITH GEOMETRY...

Deduction

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Identify angles in parallel lines
- Solve angle problems
- Make conjectures with angles
- Make conjectures with shapes

Keywords

Parallel: two straight lines that never meet with the same gradient.
Perpendicular: two straight lines that meet at 90°
Transversal: a line that crosses at least two other lines.
Sum: the result of adding two or more numbers.
Conjecture: a statement that might be true but is not proven.
Equation: a statement that says two things are equal
Polygon: a 2D shape made from straight edges.
Counterexample: an example that disproves a statement

Alternate angles

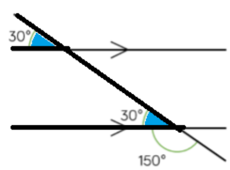
R

Because alternate angles are equal the highlighted angles are the same size

Corresponding angles

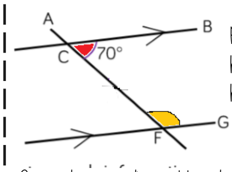
R

Because corresponding angles are equal the highlighted angles are the same size



Co-interior angles

R



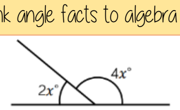
Because co-interior angles have a sum of 180° the highlighted angle is 110°

As angles on a line add up to 180° co-interior angles can also be calculated from applying alternate/ corresponding rules first

Solving angle problems

Angles on a straight Line
180°

Link angle facts to algebra



Form an equation

$2x + 4x = 180°$

State the reason

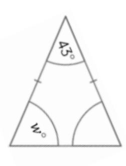
The sum of angles on a straight line is 180°

Solve

$$2x + 4x = 180°$$
$$6x = 180°$$
$$x = 30°$$

Vertically opposite angles
Equal

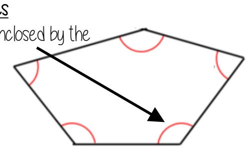
Angles around a point
360°



Triangles
Sum of angles is 180°

Isosceles have the same base angles

Interior Angles
The angles enclosed by the polygon



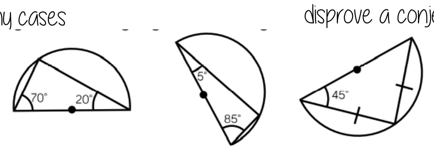
(number of sides – 2) x 180

Making conjectures with angles

True | Always | Never | False | Sometimes

Proving a conjecture
A pattern is noticed for many cases

Disproving a conjecture
Only one counterexample is needed to disprove a conjecture



Apply the angle rules

The sum of angles in a triangle is 180°

Test the theory

$180 - 70 - 20 = 90$
$180 - 85 - 5 = 90$
$180 - 45 - 45 = 90$

Make conjecture

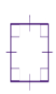
The angle that meets the circumference in a semi circle is 90°

Making conjectures with shapes

Keywords and facts to recall with shape

Area: the amount of space inside a shape
Perimeter: the length around a shape
Regular Polygons: All sides and angles are equal

Quadrilateral Facts



Square
All sides equal size
All angles 90°
Opposite sides are parallel

Parallelogram
Opposite sides are parallel
Opposite angles are equal
Co-interior angles

Rectangle
All angles 90°
Opposite sides are parallel

Kite
No parallel lines
Equal lengths on top sides
Equal lengths on bottom sides
One pair of equal angles

Rhombus
All sides equal size
Opposite angles are equal

YEAR 9 – REASONING WITH GEOMETRY...
Rotation & Translation

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

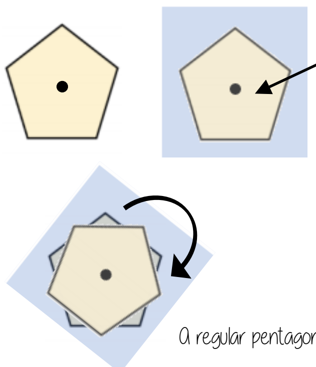
- Identify the order of rotational symmetry
- Rotate a shape about a point on the shape
- Rotate a shape about a point not on a shape
- Translate by a given vector
- Compare rotations and reflections

Keywords

Rotate: a rotation is a circular movement.
Symmetry: when two or more parts are identical after a transformation.
Regular: a regular shape has angles and sides of equal lengths.
Invariant: a point that does not move after a transformation.
Vertex: a point two edges meet.
Horizontal: from side to side
Vertical: from up to down

Rotational Symmetry

Tracing paper helps check rotational symmetry



1. Trace your shape (mark the centre point)
2. Rotate your tracing paper on top of the original through 360°
3. Count the times it fits back into itself

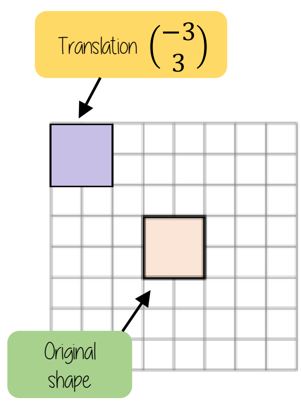
a regular pentagon has rotational symmetry of order 5

Translation and vector notation

Vector Notation → $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$

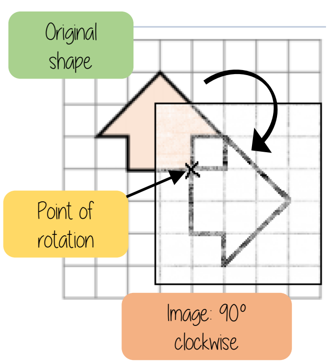
How far left or right to move
Negative value (left)
Positive value (right)

How far up or down to move
Negative value (down)
Positive value (up)

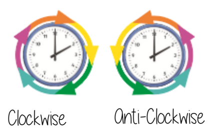


Every vertex has been translated by the same amount

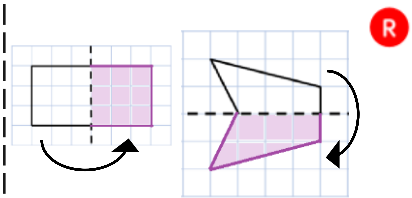
Rotate from a point (in a shape)



1. Trace the original shape (mark the point of rotation)
2. Keep the point in the same place and turn the tracing paper
3. Draw the new shape

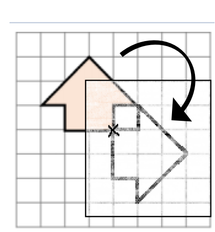


Compare rotations and reflections



Reflections are a mirror image of the original shape.

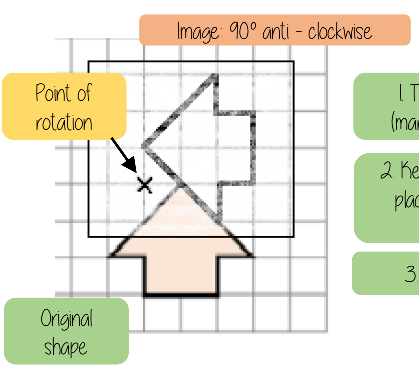
Information needed to perform a reflection:
- Line of reflection (Mirror line)



Rotations are the movement of a shape in a circular motion

Information needed to perform a rotation:
- Point of rotation
- Direction of rotation
- Degrees of rotation

Rotate from a point (outside a shape)



1. Trace the original shape (mark the point of rotation)
2. Keep the point in the same place and turn the tracing paper
3. Draw the new shape

YEAR 9 – REASONING WITH GEOMETRY...

Pythagoras' theorem

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Use square and cube roots
- Identify the hypotenuse
- Calculate the hypotenuse
- Find a missing side in a Right angled triangle
- Use Pythagoras' theorem on axes
- Explore proofs of Pythagoras' theorem

Keywords

Square number: the output of a number multiplied by itself

Square root: a value that can be multiplied by itself to give a square number

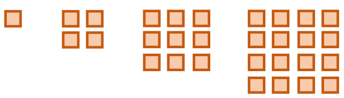
Hypotenuse: the largest side on a right angled triangle. Always opposite the right angle.

Opposite: the side opposite the angle of interest

Adjacent: the side next to the angle of interest

Squares and square roots

R



This can also be written as 6^2

$\sqrt{\ }$ is the square root symbol

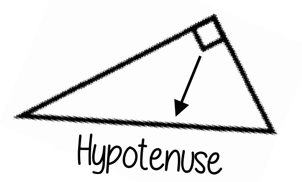
eg $\sqrt{64} = 8$

Because 8 × 8 = 64

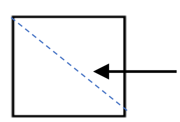
1 × 1	2 × 2	3 × 3	4 × 4	5 × 5	6 × 6	7 × 7	8 × 8	9 × 9	10 × 10
1	4	9	16	25	36	49	64	81	100

Square numbers

Identify the hypotenuse

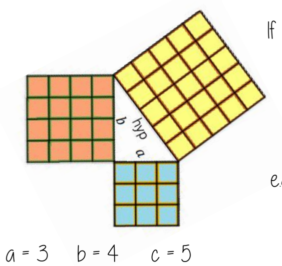


The hypotenuse is always the longest side on a triangle because it is opposite the biggest angle.



Polygons can still have a hypotenuse if it is split up into triangles and opposite a right angle

Determine if a triangle is right-angled



If a triangle is right-angled, the sum of the squares of the shorter sides will equal the square of the hypotenuse.

$a^2 + b^2 = \text{hypotenuse}^2$

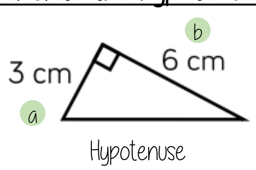
eg $a^2 + b^2 = \text{hypotenuse}^2$

$3^2 + 4^2 = 5^2$

$9 + 16 = 25$

Substituting the numbers into the theorem shows that this is a right-angled triangle

Calculate the hypotenuse



Either of the short sides can be labelled a or b

$a^2 + b^2 = \text{hypotenuse}^2$

1. Substitute in the values for a and b

$3^2 + 6^2 = \text{hypotenuse}^2$

$9 + 36 = \text{hypotenuse}^2$

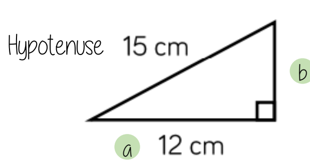
$45 = \text{hypotenuse}^2$

2. To find the hypotenuse square root the sum of the squares of the shorter sides.

$\sqrt{45} = \text{hypotenuse}$

$6.71cm = \text{hypotenuse}$

Calculate missing sides



Either of the short sides can be labelled a or b

$a^2 + b^2 = \text{hypotenuse}^2$

$12^2 + b^2 = 15^2$

1. Substitute in the values you are given

$144 + b^2 = 225$

$-144 \qquad -144$

Rearrange the equation by subtracting the shorter square from the hypotenuse squared

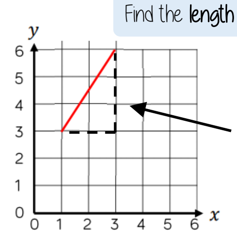
Square root to find the length of the side

$b^2 = 111$

$b = \sqrt{111} = 10.54\ cm$

Pythagoras' theorem on a coordinate axis

Find the **length** of the line segment



The segment can be made into a right-angled triangle by adding the sides on the diagram.

The line segment is the **hypotenuse**

$a^2 + b^2 = \text{hypotenuse}^2$

The lengths of a and b are the sides of the triangle.

Be careful to check the scale on the axes

YEAR 9 – REASONING WITH GEOMETRY…

Enlargement & Similarity

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Recognise enlargement and similarity
- Enlarge a shape by a positive SF
- Enlarge a shape from a point
- Enlarge a shape by a fractional SF
- Work out missing sides and angles in a pair of similar shapes.

Keywords

Similar Shapes: shapes of different sizes that have corresponding sides in equal proportion and identical corresponding angles.

Scale Factor: the multiple describing how much a shape has been enlarged

Enlarge: to change the size of a shape (enlargement is not always making a shape bigger)

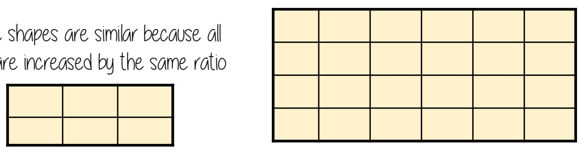
Corresponding: objects (or sides) that appear in the same place in two similar situations.

Image: the picture or visual representation of the shape

Recognise enlargement & similarity

Shapes are similar if all pairs of corresponding sides are in the same ratio

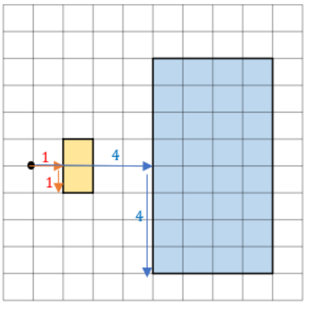
These shapes are similar because all sides are increased by the same ratio



Enlargements are similar shapes with a ratio other than 1

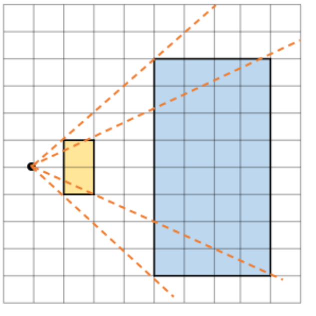
Enlarge a shape from a point

Scaled distances method



Scale the distance between the point of enlargement and each corresponding vertices

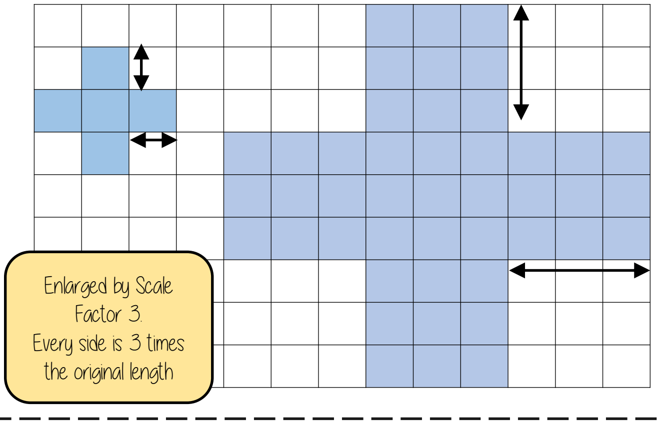
Rays method



Multiply the distance from the centre of corresponding vertices by the scale factor along the ray

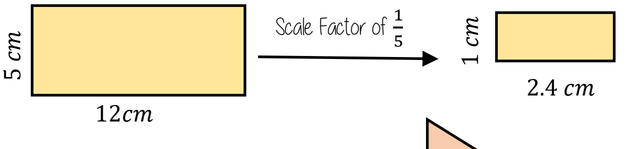
Enlarge by a positive scale factor

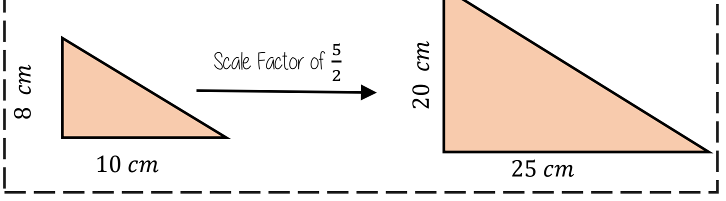
With a scale factor larger than 1 it makes the shape **bigger**



Positive fractional scale factor

With a scale factor between 0 and 1 it makes the shape **smaller**

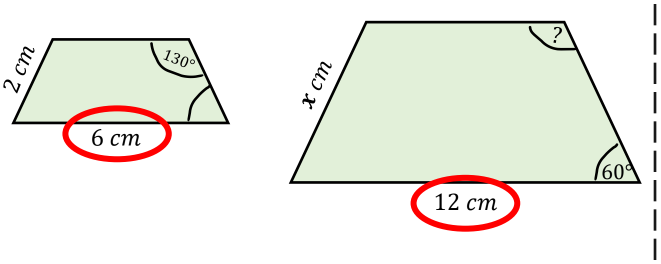




Calculations in similar shapes

Don't forget that properties of shapes don't change with enlargements or in similar shapes

The two trapezium are similar find the missing side and angle



<u>Corresponding sides identify the scale factor</u> $\frac{12}{6} = 2$ Scale Factor = **2**

<u>Calculate the missing side</u> Length (corresponding side) x scale factor

$2cm \times 2$

$x = 4cm$

Enlargement does not change angle size

<u>Calculate the missing angle</u> Corresponding angles remain the same

$130°$

YEAR 9 – REASONING WITH GEOMETRY...

Solving ratio & proportion problems

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Solve problems with direct proportion
- Use conversion graphs
- Solve problems with inverse proportion
- Solve ratio problems
- Solve 'best buy' problems

Keywords

Proportion: a comparison between two numbers

Ratio: a ratio shows the relative size of two variables

Direct proportion: as one variable is multiplied by a scale factor the other variable is multiplied by the same scale factor.

Inverse proportion:: as one variable is multiplied by a scale factor the other is divided by the same scale factor.

Direct Proportion

As one variable changes the other changes at the same rate.

R

4 cans of pop = £2.40

This is a multiplicative change

4 cans of pop = £2.40 (× 0.5)

2 cans of pop = £1.20 (× 0.5)

This multiplier is the same In the same way that this would be for ratio

4 cans of pop = £2.40 (× 3)

12 cans of pop = £7.20 (× 3)

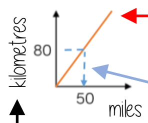
Sometimes this is easiest if you work out how much one unit is worth first

e.g. 1 can of pop = £0.60

Conversion Graphs

Compare two variables

R



This is always a straight line because as one variable increases so does the other at the same rate

Labelling of both axes is vital

To make conversions between units you need to find the point to compare – then find the associated point by using your graph.

Using a ruler helps for accuracy

Showing your conversion lines help as a "check" for solutions

Inverse Proportion

As one variable is multiplied by a scale factor the other is divided by the same scale factor

Examples of inversely proportional relationships

Time taken to fill a pool and the number of taps running

Time taken to paint a room and the number of workers

T is inversely proportional to G. When T=2 then G=20

÷ 2 × 4

T	1	2	8
G	40	20	5

× 2 ÷ 4

Best Buys

Have a directly proportional relationship

To calculate best buys you need to be able to compare the cost of one unit or units of equal amounts

Shop A

4 cans for £1.20

£1.20 ÷ 4

1 can is £0.30 Or 30p

Shop B

3 cans for 93p

£0.93 ÷ 3

1 can is £0.31 Or 31p

Cost per item

Shop **A** is the best value as it is 1p cheaper per can of pop

Shop A

4 cans for £1.20

4 ÷ £1.20

£1 buys 3.333 cans of pop

3 cans for 93p

3 ÷ £0.93

£1 buys 3.23 cans of pop

Cost per pound

Shop **A** is still shown as being the best value but pay attention to the unit you are calculating, per item or per pound

Best value is the most product for the lowest price per unit

Sharing a whole into a given ratio

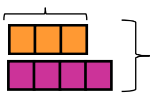
R

James and Lucy share £350 in the ratio 3:4.
Work out how much each person earns

Model the Question

James: Lucy

3 : 4



Find the value of one part

Whole: £350

7 parts to share between

(3 James, 4 Lucy)

£350 ÷ 7 = £50

□ = one part = £50

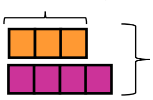
Put back into the question

James: Lucy

3 : 4 (× 50)

£150:£200

James = 3 x £50 = £150



Lucy = 4 x £50 = £200

Finding a value given 1:n (or n:1)

R

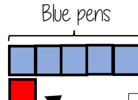
Inside a box are blue and red pens in the ratio 5:1
If there are 10 red pens how many blue pens are there?

Model the Question

Blue : Red

5 : 1

□ = one part = 10 pens



One unit = 10 pens

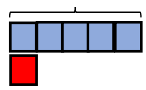
Put back into the question

Blue : Red

5 : 1 (x 10)

50: 10

Blue pens = 5 x 10 = 50 pens



Red pens = 1 x 10 = 10 pens

There are 50 Blue Pens

YEAR 9 – REASONING WITH GEOMETRY… Rates

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Solve speed, distance, time questions
- Use distance time graphs
- Solve density, mass, volume problems
- Solve flow problems
- Use flow graphs
- Interpret rates of change and their units

Keywords

Convert: change

Mass: a measure of how much matter is in an object. Commonly measured by weight.

Origin: the coordinate (0, 0)

Volume: the amount of 3D space a shape takes up

Substitute: putting numbers where letters are – replacing numbers into a formula

Speed, Distance, Time

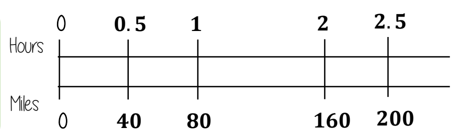
"per" for every

e.g. 80 miles per hour (mph)

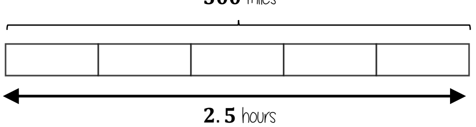
Travel 80 miles every hour

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

You can use a double number line to help you calculate distance



e.g. A boat travels at a constant speed for 2.5 hours. It travels 300 miles.



Bar models can help to calculate mph

Each part is half an hour

Each part is 60 miles

Speed, Distance, Time

Before calculations – make sure you are working in the same units as the speed

minutes → hours: ÷ 60

hours → minutes: × 60

Learn or learn how to rearrange the formular for speed, distance and time

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

$$\text{distance} = \text{speed} \times \text{time}$$

Substitute in the variables given

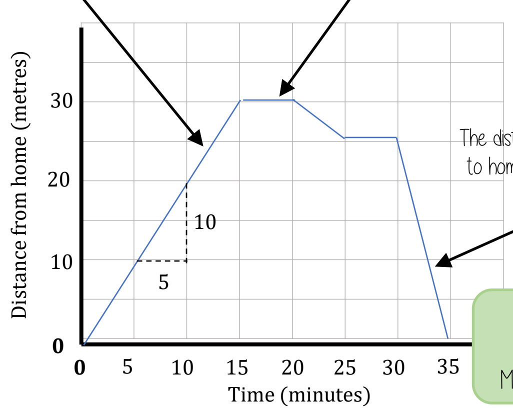
Distance – Time graphs

The steeper a gradient the faster the speed

$\frac{10}{5} = 2$ metres per min

Gradient = speed

Horizontal lines represent staying still



The distance coming closer to home shows the return journey

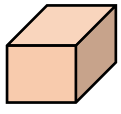
Units are important. Meters per minute

Density, Mass, Volume

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

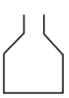
$$\text{volume} = \frac{\text{mass}}{\text{density}}$$

$$\text{mass} = \text{volume} \times \text{density}$$

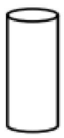


volume of prism = Area of cross section × Depth

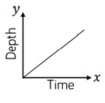
Flow problems & graphs



This will fill at a constant rate, then as the space decreases it will speed up and the neck of the bottle fill at a faster constant speed



The cylinder will fill at a constant speed



Units are important. Ensure any volume calculations are the same unit as the rate of flow

Rates of change & units

Common rates of change relationships

Revisit your conversions between units of length and capacity

Speed: miles **per** hour

Exchange rates: euros **per** pounds

Density: mass **per** volume

metres → kilometres: ÷ 1000

kilometres → metres: × 1000

YEAR 9 – REPRESENTATIONS…

Probability

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

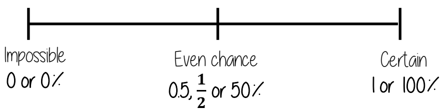
- Find single event probability
- Find relative frequency
- Find expected outcomes
- Find independent events
- Use diagrams to work out probabilities

Keywords

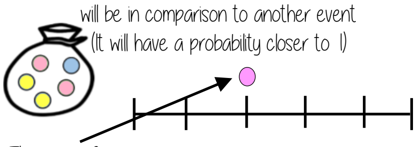
Probability: the chance that something will happen
Relative Frequency: how often something happens divided by the outcomes
Independent: an event that is not effected by any other events.
Chance: the likelihood of a particular outcome.
Event: the outcome of a probability – a set of possible outcomes.
Biased: a built in error that makes all values wrong by a certain amount.

The probability scale

R



The more likely an event the further up the probability it will be in comparison to another event (It will have a probability closer to 1)



There are 2 pink and 2 yellow balls, so they have the same probability

There are 5 possible outcomes
So 5 intervals on this scale, each interval value is $\frac{1}{5}$

Single event probability

R

Probability is always a value between 0 and 1

The probability of getting a blue ball is $\frac{1}{5}$
∴The probability of NOT getting a blue ball is $\frac{4}{5}$

The sum of the probabilities is 1

The table shows the probability of selecting a type of chocolate

Dark	Milk	White
0.15	0.35	

P(white chocolate) = 1 – 0.15 – 0.35
= 0.5

Relative Frequency

$$\frac{\text{Frequency of event}}{\text{Total number of outcomes}}$$

Remember to calculate or identify the overall number of outcomes!

Colour	Frequency	Relative Frequency
Green	6	0.3
Yellow	12	0.6
Blue	2	0.1
	20	

Relative frequency can be used to find expected outcomes

e.g. Use the relative probability to find the expected outcome for green if there are 100 selections.

Relative frequency x Number of times
$0.3 \times 100 = 30$

Expected outcomes

Expected outcomes are estimations. It is a long term average rather than a prediction.

Dark	Milk	White
0.15	0.35	0.5

The sum of the probabilities is 1

An experiment is carried out 400 times.
Show that dark chocolate is expected to be selected 60 times

$0.15 \times 400 = 60$

Independent events

The rolling of one dice has no impact on the rolling of the other. The individual probabilities should be calculated separately.

Probability of event 1 × Probability of event 2

Find the probability of getting a 5 and a red

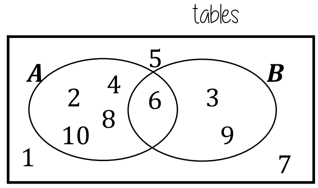
$P(5) = \frac{1}{6}$ $P(R) = \frac{1}{4}$

$P(5 \ and \ R) = \frac{1}{6} \times \frac{1}{4} = \frac{1}{24}$

Using diagrams

Recap Venn diagrams, Sample space diagrams and Two-way tables

R



	Car	Bus	Walk	Total
Boys	15	24	14	53
Girls	6	20	21	47
Total	21	44	35	100

The possible outcomes from rolling a dice

The possible outcomes from tossing a coin

	1	2	3	4	5	6
H	1,H	2,H	3,H	4,H	5,H	6,H
T	1,T	2,T	3,T	4,T	5,T	6,T

YEAR 9 — REPRESENTATIONS...

Algebraic Representation

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Draw quadratic graphs
- Interpret quadratic graphs
- Interpret other graphs including reciprocals
- Represent inequalities

Keywords

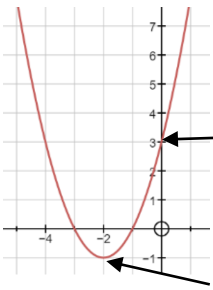
Quadratic: a curved graph with the highest power being 2. Square power.
Inequality: makes a non equal comparison between two numbers
Reciprocal: a reciprocal is 1 divided by the number
Cubic: a curved graph with the highest power being 3. Cubic power.
Origin: the coordinate (0, 0)
Parabola: a 'u' shaped curve that has mirror symmetry

Quadratic Graphs

$y = x^2 + 4x + 3$

If x^2 is the highest power in your equation then you have a quadratic graph.

It will have a parabola shape



Intersection with the y axis

Substitute the x values into the equation of your line to find the y coordinates

x	-4	-3	-2	-1	0	1
y	3	0	-1	0	3	8

Coordinate pairs for plotting $(-3, 0)$

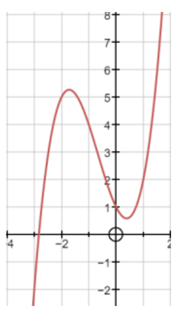
Plot all of the coordinate pairs and join the points with a curve (freehand)

Quadratic graphs are always symmetrical with the turning point in the middle

Interpret other graphs

Cubic Graphs

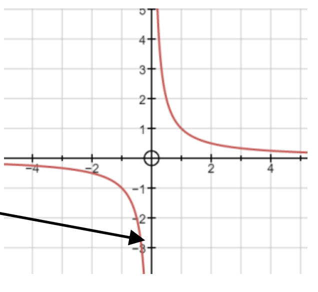
$y = x^3 + 2x^2 - 2x + 1$



If x^3 is the highest power in your equation then you have a cubic graph

Reciprocal Graphs

$y = \frac{1}{x}$

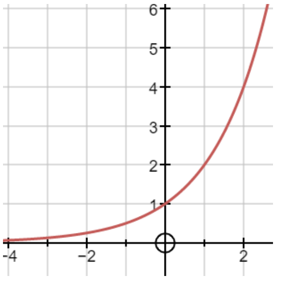


Reciprocal graphs never touch the y axis.
This is because x cannot be 0
This is an asymptote

Exponential Graphs

$y = 2^x$

Exponential graphs have a power of x

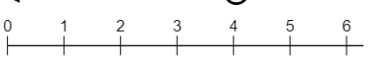


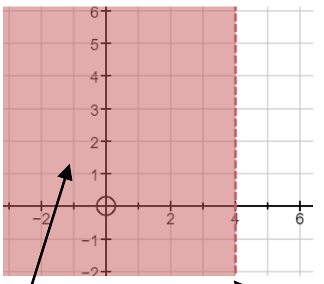
Represent Inequalities

Multiple methods of representing inequalities

$x < 4$

All values are less than 4



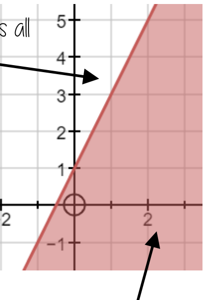


The shaded area indicates all possible values of x

The dotted line shows that the inequality does not include these points

The solid line shows that the inequality includes all the points on this line

$y \geq 2x + 1$



The shaded area indicates all possible solutions to this inequality