YEAR 8 - PROPORTIONAL REASONING...

Ratio and Scale

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Simplify any given ratio
- Share an amount in a given ratio
- Solve ratio problems given a part

Solutions should be modelled, explained and solved.

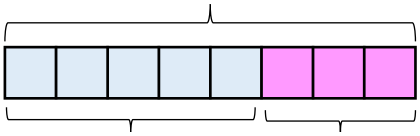
Keywords

Ratio: a statement of how two numbers compare
Equal Parts:: all parts in the same proportion, or a whole shared equally
Proportion: a statement that links two ratios
Order: to place a number in a determined sequence
Part: a section of a whole
Equivalent: of equal value
Factors: integers that multiply together to get the original value
Scale: the comparison of something drawn to its actual size.

Representing a ratio

"For every 5 boys there are 3 girls"

This is the "whole" – boys and girls together

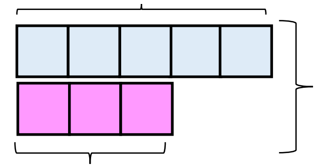


This represents the 5 boys | This represents the 3 girls

5:3

Double Number Line

This represents the 5 boys



This is the "whole" – boys and girls together

This represents the 3 girls

Order is Important

"For every dog there are 2 cats"

Dogs: Cats

1:2

The ratio has to be written in the same order as the information is given.

e.g. 2:1 would represent 2 dogs for every 1 cat. ✕

Simplifying a ratio

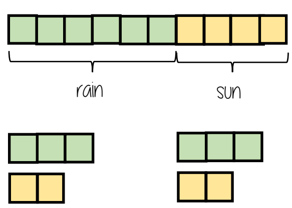
Cancel down the ratio to its lowest form

"For every 6 days of rain there are 4 days of sun"

6:4

÷ by 2

3:2



Find the biggest common factor that goes into all parts of the ratio

For 6 and 4 the biggest factor (number that multiples into them is 2)

"For every 3 days of rain there are 2 days of sun" – when this happens twice the ratio becomes 6:4.

Ratio 1:n (or n:1)

This is asking you to cancel down until the part indicated represents 1

Show the ratio 4:20 in the ratio of 1:n

The question states that **this part has to be 1 unit** Therefore **Divide by 4**

4 : 20

1 : 5

This side has to be divided by 4 too – to keep in **proportion**

H the n part does not have to be an integer for this type of question

Units are important:

When using a ratio – all parts should be in the same units

Useful Conversions

mm → cm (x10), cm → mm (÷10); cm → m (x100), m → cm (÷100); m → km (x1000), km → m (÷1000)

g → kg (x1000), kg → g (÷1000)

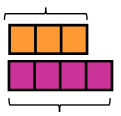
ml → L (x1000), L → ml (÷1000)

Sharing a whole into a given ratio

James and Lucy share £350 in the ratio 3:4.
Work out how much each person earns

Model the Question

James: Lucy
3 : 4



Find the value of one part

Whole: £350
7 parts to share between
(3 James, 4 Lucy)

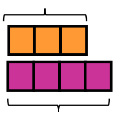
£350 ÷ 7 = £50

☐ = one part = £50

Put back into the question

James: Lucy
3 : 4 (x 50, x 50)
£150:£200

James = 3 x £50 = £150



Lucy = 4 x £50 = £200

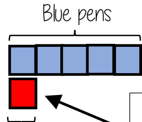
Finding a value given 1:n (or n:1)

Inside a box are blue and red pens in the ratio 5:1
If there are 10 red pens how many blue pens are there?

Model the Question

Blue : Red
5 : 1

☐ = one part = 10 pens

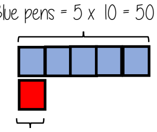


One unit = 10 pens

Put back into the question

Blue : Red
5 : 1 (x 10, x 10)
50: 10

Blue pens = 5 x 10 = 50 pens

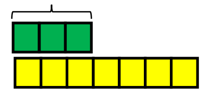


Red pens = 1 x 10 = 10 pens

There are 50 Blue Pens

Ratio as a fraction

Trees: Flowers
3 : 7



Ratio

There are 3 parts for trees

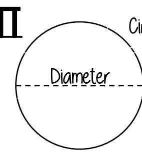
Fraction of trees

$\frac{\text{Number of parts of in group}}{\text{Total number of parts}}$ → $\frac{3}{10}$

Fraction

Tree parts 3 + Flower parts 7 = 10

Pi π



The ratio of a circles circumference to its diameter

YEAR 8 - PROPORTIONAL REASONING...

Multiplicative Change

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Solve problems and explain direct proportion
- Use conversion graphs to make statements, comparisons and form conclusions.
- Understand and use scale factors for length

Keywords

Proportion: a statement that links two ratios
Variable: a part that the value can be changed
Axes: horizontal and vertical lines that a graph is plotted around
Approximation: an estimate for a value
Scale Factor: the multiple that increases/ decreases a shape in size
Currency: the system of money used in a particular country
Conversion: the process of changing one variable to another
Scale: the comparison of something drawn to its actual size.

Direct Proportion

As one variable changes the other changes at the same rate.

4 cans of pop = £2.40

This is a multiplicative change

4 cans of pop = £2.40 (x 3) → 12 cans of pop = £7.20 (x 3)

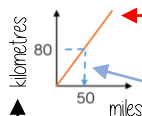
4 cans of pop = £2.40 (x 0.5) → 2 cans of pop = £1.20 (x 0.5)

This multiplier is the same In the same way that this would be for ratio

Sometimes this is easiest if you work out how much one unit is worth first
e.g. 1 can of pop = £0.60

Conversion Graphs

Compare two variables



This is always a straight line because as one variable increases so does the other at the same rate

Labelling of both axes is vital

To make conversions between units you need to find the point to compare – then find the associated point by using your graph.
Using a ruler helps for accuracy
Showing your conversion lines help as a "check" for solutions

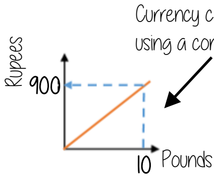
Conversion between currencies

£1 = 90 Rupees ← Currency is directly proportional

For every £1 I have 90 Rupees

£1 = 90 Rupees (x 10) → £10 = 900 Rupees (x 10)

Currency can be converted using a conversion graph

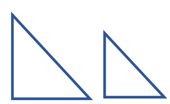


Convert 630 Rupees into Pounds

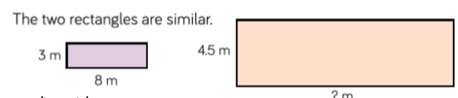
630 ÷ 90 = 7

£1 = 90 Rupees (x 7) → £7 = 630 Rupees (x 7)

Ratio between similar shapes



Angles in similar shapes do not change. e.g. if a triangle gets bigger the angles can not go above 180°



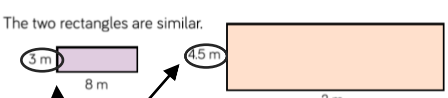
Corresponding sides

3m : 4.5m (÷ 3) → 1 m : 1.5 m (÷ 3)

8m : 12m (÷ 8) → 1 m : 1.5 m (÷ 8)

Note
Simplify to the same ratio

Understand Scale Factor



3 x 1.5 = 4.5

This is a multiplicative change

Use **corresponding** sides to calculate a scale factor

Missing length
8 x 1.5 = 12m

Scale factor can also be calculated by:

Bigger corresponding side / **Smaller corresponding side**

Small corresponding side → x SF → Big corresponding side

Big corresponding side → ÷ SF → Small corresponding side

Draw and interpret scale diagrams

A picture of a car is drawn with a scale of 1:30

For every 1cm on my image is 30cm in real life

The car image is 10cm

Image : Real life
1cm : 30cm (x 10)
10cm : 300cm (x 10)

The car in real life is 210cm

Image : Real life
1cm : 30cm (x 7)
7cm : 210 cm (x 7)

Interpret maps with scale factors

mm → ÷ 10 → cm → ÷ 100 → m → ÷ 1000 → km

km → x 1000 → m → x 100 → cm → x 10 → mm

1 cm : 250 m

Ratios need to be in the same units

1 cm : 250m

250 x 100 = 25000

1 cm : 25000cm

For every 1cm on my map is 25000cm in real life.

YEAR 8 - PROPORTIONAL REASONING...

Multiplying and Dividing Fractions

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Carry out any multiplication or division using fractions and integers.
- Solutions can be modelled, described and reasoned

Keywords

Numerator: the number above the line on a fraction. The top number. Represents how many parts are taken

Denominator: the number below the line on a fraction. The number represent the total number of parts.

Whole: a positive number including zero without any decimal or fractional parts.

Commutative: an operation is commutative if changing the order does not change the result.

Unit Fraction: a fraction where the numerator is one and denominator a positive integer.

Non-unit Fraction: a fraction where the numerator is larger than one.

Dividend: the amount you want to divide up.

Divisor: the number that divides another number.

Quotient: the answer after we divide one number by another. e.g. dividend ÷ divisor = quotient

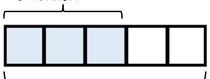
Reciprocal: a pair of numbers that multiply together to give 1.

Representing a fraction

Numerator / Denominator

$\frac{3}{5}$

Number of parts represented – Numerator



Number of parts to make up the whole – Denominator

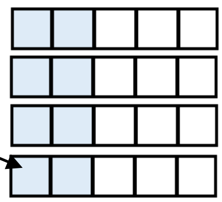
ALL PARTS of a fraction are of equal size

Repeated addition = multiplication by an integer

$4 \times \frac{2}{5} \rightarrow \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5}$

Integer (Whole number)

Each part represents $\frac{1}{5}$

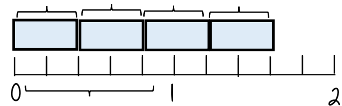


How many parts are shaded? / What each part represents

Revisit: When adding fractions with the same denominator = add the numerators

$= \frac{8}{5} \leftarrow \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5}$

$= 1\frac{3}{5}$



Each whole is split into the same number of parts as the denominator

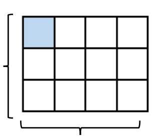
Multiplying unit fractions

$\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$

Parts shaded

Total number of parts in the diagram

Modelled:



Multiplying non-unit fractions

$\frac{3}{4} \times \frac{2}{3}$

Shade in 3 parts

Repeat it on this many rows

This many columns

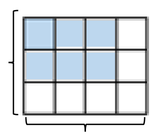
This many rows

$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$

Parts shaded

Total number of parts in the diagram

Modelled:



Quick Multiplying and Cancelling down

$\frac{\cancel{3}^{1}}{5} \times \frac{4}{\cancel{9}_{3}}$

The 3 and the 9 have a common factor and can be simplified

Quick Solving

Multiply the numerators

Multiply the denominators

$\frac{1 \times 4}{5 \times 3} = \frac{4}{15}$

The reciprocal

When you multiply a number by its reciprocal the answer is always 1

$3 \times \frac{1}{3} = 1$

$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$

The reciprocal of 3 is $\frac{1}{3}$ and vice versa

Reciprocals for division

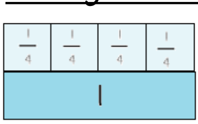
e.g.

$5 \div \frac{1}{4} = 20$

$5 \times 4 = 20$

Multiplying by a reciprocal gives the same outcome

Dividing an integer by an unit fraction



$1 \div \frac{1}{4} = 4$

How many quarters are in 1?

"There are 4 quarters in 1 whole. Therefore, there are 20 quarters in 5 wholes"

$5 \div \frac{1}{4} = 20$

Dividing any fractions

Remember to use reciprocals

$\frac{2}{5} \div \frac{3}{4}$

$\frac{2}{5} \times \frac{4}{3}$

Multiplying by a reciprocal gives the same outcome

Represented

$= \frac{8}{15}$

YEAR 8 - REPRESENTATIONS...

Working in the Cartesian plane

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Label and identify lines parallel to the axes
- Recognise and use basic straight lines
- Identify positive and negative gradients
- Link linear graphs to sequences
- Plot y = mx + c graphs

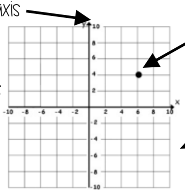
Keywords

Quadrant: four quarters of the coordinate plane.
Coordinate: a set of values that show an exact position.
Horizontal: a straight line from left to right (parallel to the x axis)
Vertical: a straight line from top to bottom (parallel to the y axis)
Origin: (0,0) on a graph. The point the two axes cross
Parallel: Lines that never meet
Gradient: The steepness of a line
Intercept: Where lines cross

Coordinates in four quadrants

y-axis

x-axis



Coordinate (x, y) (6, 4)

From the origin this coordinate is 6 places along the positive x axis and 4 places up the positive y axis.

(x, y)

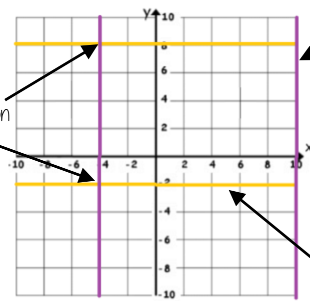
Always the position on the x axis first

Always the position on the y axis second

(0, a) Will be always be a point on the y axis (a can be any number)

(a, 0) Will be always be a point on the x axis (a can be any number)

Lines parallel to the axes



Intersection points

All the points on this line have a x coordinate of 10

'a' can be ANY positive or negative value including 0

Lines parallel to the **y axis** take the form **x = a** and are **vertical**

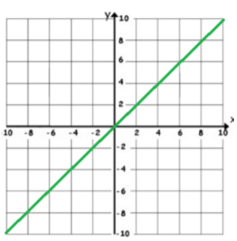
Lines parallel to the **x axis** take the form **y = a** and are **horizontal**

All the points on this line have a y coordinate of -2

e.g (3, -2) (7, -2) (-2, -2) all lay on this line because the y coordinate is -2

Recognise and use the line y=x

This means the x and the y coordinate have the same value

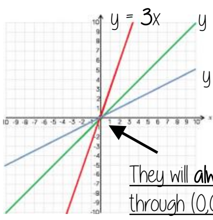


Examples of coordinates on this line: (0, 0) (-3, -3) (8, 8)

The axes **scale is important** – if the scale is the same y = x will be a straight line at 45°

Recognise and use the lines y=kx

The value of **k** changes the steepness of the line



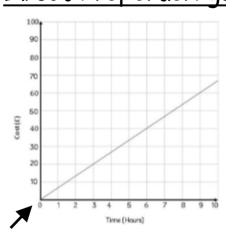
Note: y = x is the same as y = 1x

They will **always** go through (0,0)

The bigger the value of k the **steeper** the line will be.

The closer to 0 the value of k the closer the line will be to the x axis.

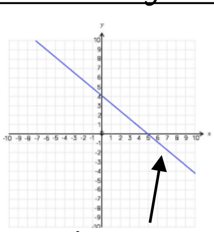
Direct Proportion using y=kx



The line must be straight to be directly proportional – variables increase at the same rate **k**

Direct proportion graphs always start at (0,0) as they are describing relationships between two variables

Lines with negative gradients

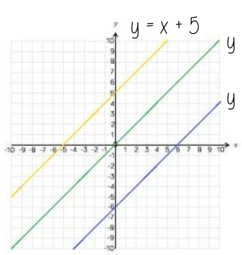


Any straight-line graph with a negative x value has a negative gradient.

Eg y = -2x
y = -x   y + x = 12

Direction of all negative gradients

Lines in the form y = x + a



All the lines are **parallel** because the gradients are the same

y = x + a

This is the line y=x when the y and x coordinate are the same

This shows the translation of that line.
e.g y = x + 5
Is the line y=x moved 5 places up the graph

5 has been added to each of the x coordinates

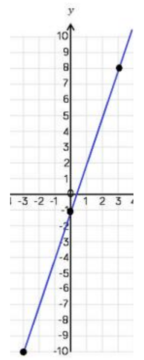
Plotting y = mx + c graphs

$y = 3x - 1$ → 3 x the x coordinate then – 1

x	-3	0	3
y	-10	-1	8

Draw a table to display this information

This represents a coordinate pair (-3, -10)



You only need two points to form a straight line

Plotting more points helps you decide if your calculations are correct (if they do make a straight line)

Remember to join the points to make a line

YEAR 8 - REPRESENTATIONS...

Representing Data

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Draw and interpret scatter graphs
- Describe correlation and relationships
- Identify different types of non-linear relationships
- Design and complete an ungrouped frequency table
- Read and interpret grouped tables (discrete and continuous data)
- Represent data in two way tables

Keywords

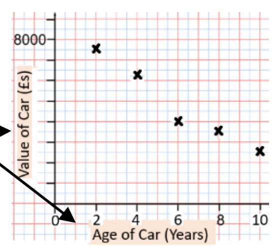
Variable: a quantity that may change within the context of the problem.
Relationship: the link between two variables (items). E.g. Between sunny days and ice cream sales
Correlation: the mathematical definition for the type of relationship.
Origin: where two axes meet on a graph.
Line of best fit: a straight line on a graph that represents the data on a scatter graph.
Outlier: a point that lies outside the trend of graph.
Quantitative: numerical data
Qualitative: descriptive information, colours, genders, names, emotions etc.
Continuous: quantitative data that has an infinite number of possible values within its range.
Discrete: quantitative or qualitative data that only takes certain values.
Frequency: the number of times a particular data value occurs.

Draw and interpret a scatter graph

Age of Car (Years)	2	4	6	8	10
Value of Car (£s)	7500	6250	4000	3500	2500

- This data may not be given in size order
- The data forms information pairs for the scatter graph
- Not all data has a relationship

All axes should be labelled

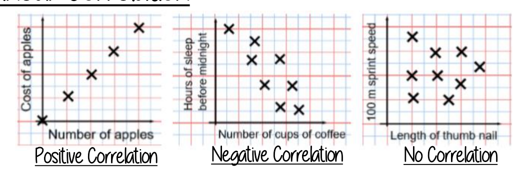


"This scatter graph show as the age of a car increases the value decreases"

The link between the data can be explained verbally

The axis should fit all the values on and be equally spread out

Linear Correlation



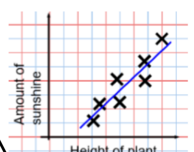
Positive Correlation	Negative Correlation	No Correlation
As one variable increases so does the other variable	As one variable increases the other variable decreases	There is no relationship between the two variables

The line of best fit

The Line of best fit is used to make estimates about the information in your scatter graph

Things to know:

- The line of best fit DOES NOT need to go through the origin (The point the axes cross)
- There should be approximately the same number of points above and below the line (It may not go through any points)
- The line extends across the whole graph



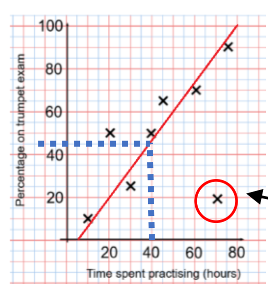
It is only an estimate because the line is designed to be an average representation of the data

It is always a straight line.

Using a line of best fit

Interpolation is using the line of best fit to estimate values inside our data point.

e.g. 40 hours revising predicts a percentage of 45.



Extrapolation is where we use our line of best fit to predict information outside of our data.
This is not always useful – in this example you cannot score more that 100%. So revising for longer can not be estimated

This point is an "**outlier**"
It is an outlier because it doesn't fit this model and stands apart from the data

Ungrouped Data

The number of times an event happened

The table shows the number of siblings students have. The answers were
3, 1, 2, 2, 0, 3, 4, 1, 1, 2, 0, 2

2 people had 0 siblings. This means there are 0 siblings to be counted here

Number of siblings	Frequency	
0	2	0
1	3	3
2	4	2 + 2 + 2 + 2 OR 2 x 4 = 8
3	2	3 + 3 OR 3 x 2 = 6
4	1	4

Best represented by discrete data (Not always a number)

2 people have 3 siblings so there are 6 siblings in total

OVERALL there are
0 + 3 + 8 + 6 + 4
Siblings = 21 siblings

Grouped Data

If we have a large spread of data it is better to group it. This is so it is easier to look for a trend. Form groups of equal size to make comparison more valid and spread the groups out from the smallest to the largest value.

Discrete Data
The groups do not overlap

Cost of TV (£)	Tally	Frequency
101 – 150	𝍸 II	7
151 – 200	𝍸 𝍸 I	11
201 – 250	𝍸	5
251 - 300	III	3

We do not know the exact value of each item in a group – so an estimate would be used to calculate the overall total (Midpoint)

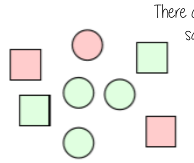
Continuous Data
To make sure all values are included inequalities represent the subgroups

x Weight(g)	Frequency
$40 < x \leq 50$	1
$50 < x \leq 60$	3
$60 < x \leq 70$	5

e.g. this group includes every weight bigger that 60Kg, up to and including 70Kg

Representing data in two-way tables

Two-way tables represent discrete information in a visual way that allows you to make conclusions, find probability or find totals of sub groups



There are 2 green squares

There are 5 green shapes

	Squares	Circles	Total
Green	2	3	5
Red	2	1	3
Total	4	4	8

There are 8 items in total

Using your two-way table

To find a fraction
e.g. What fraction of the items are red? **3 red items**
but **8 items in total** $= \frac{3}{8}$

Interleaving: Use your fraction, decimal percentage equivalence knowledge

YEAR 8 - REPRESENTATIONS...
Tables and Probability

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Construct a sample space diagram.
- Systematically list outcomes.
- Find the probability from two-way tables.
- Find the probability from Venn diagrams.

Keywords

Outcomes: the result of an event that depends on probability.
Probability: the chance that something will happen.
Set: a collection of objects.
Chance: the likelihood of a particular outcome.
Event: the outcome of a probability — a set of possible outcomes.
Biased: a built in error that makes all values wrong by a certain amount.
Union: Notation '∪' meaning the set made by comparing the elements of two sets.

Construct sample space diagrams

Sample space diagrams provide a systematic way to display outcomes from events

The possible outcomes from rolling a dice

The possible outcomes from tossing a coin

	1	2	3	4	5	6
H	1,H	2,H	3,H	4,H	5,H	6,H
T	1,T	2,T	3,T	4,T	5,T	6,T

This is the set notation to list the outcomes S =

In between the { } are a;; the possible outcomes

S = { 1H, 2H, 3H, 4H, 5H, 6H, 1T, 2T, 3T, 4T, 5T, 6T}

Probability from sample space

The possible outcomes from rolling a dice

The possible outcomes from tossing a coin

	1	2	3	4	5	6
H	1,H	2,H	3,H	4,H	5,H	6,H
T	1,T	(2,T)	3,T	(4,T)	5,T	(6,T)

What is the probability that an outcome has an even number and a tails?

This is the set notation that represents the question P

P (Even number and Tails) = $\frac{3}{12}$

In between the ( ) is the event asked for

There are three even numbers with tails

There are twelve possible outcomes

Numerator: the event

Denominator: the total number of outcomes

Probability from two-way tables

	Car	Bus	Walk	Total
Boys	15	24	14	53
Girls	6	20	(21)	47
Total	21	44	35	(100)

The total number of items

P (Girl walk to school) = $\frac{21}{100}$

The event

The total in the set

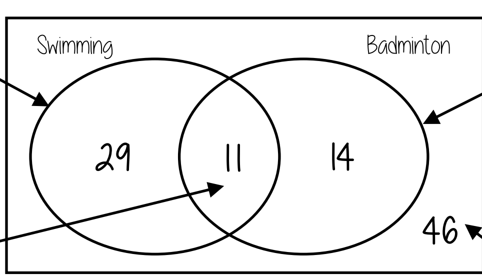
Product Rule

The number of items in event a X The number of items in event b

Probability from Venn diagrams

100 students were questioned if they played badminton or went to swimming club.
40 went swimming, 25 went to badminton and 11 went to both.

This whole curve includes everyone that went swimming.
Because 11 did both we calculate **just** swimming by 40- 11



The intersection represents both. Swimming **AND** badminton

This whole curve includes everyone that went to badminton.
Because 11 did both we calculate **just** badminton by 25 - 11

The number outside represents those that did **neither** badminton or swimming

100 – 29 – 11 – 14

P (Just swimming) = $\frac{29}{100}$

YEAR 8 - ALGEBRAIC TECHNIQUES...

@whisto_maths

Brackets, Equations & Inequalities

What do I need to be able to do?

By the end of this unit you should be able to:

- Form Expressions
- Expand and factorise single brackets
- Form and solve equations
- Solve equations with brackets
- Represent inequalities
- Form and solve inequalities

Keywords

Simplify: grouping and combining similar terms
Substitute: replace a variable with a numerical value
Equivalent: something of equal value
Coefficient: a number used to multiply a variable
Product: multiply terms
Highest Common Factor (HCF): the biggest factor (or number that multiplies to give a term)
Inequality: an inequality compares who values showing if one is greater than, less than or equal to another

Form expressions

For unknown variables, a letter is normally used in its place

More than – ADD
Less than/ difference – SUBTRACT

eg 4 more than t → $t + 4$
8 less than k → $k - 8$

Only similar terms can be grouped together

eg Find the perimeter of this shape (Perimeter = length around outside of shape)

t
2t + 1

$t + 2t + 1 + t + 2t + 1 \rightarrow 6t + 2$

Directed numbers

\+ \+ → \+
\- \- → \+
\+ \- → \-
\- \+ → \-

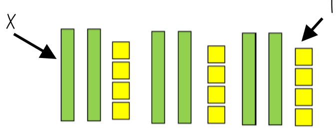
eg $a = -5$ and $b = 2$
$a^2 = a \times a = -5 \times -5 = 25$
$b + a = 2 + -5 = -3$

Multiply single brackets

$3(2x + 4)$

	2x	4
3	3 x 2x 6x	3 x 4 12

6x + 12



2x + 4	2x + 4	2x + 4
x x 4	x x 4	x x 4
6x + 12		

Different representations of $3(2x+4) = 6x + 12$

Factorise into a single bracket

$8x + 4$

8x + 4	4
2x + 1	

← Try and make this the **highest common factor**

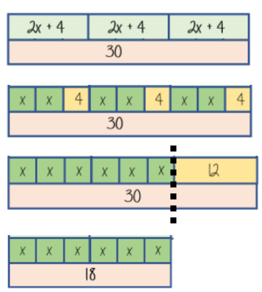
The two values **multiply** together (also the area) of the rectangle

$8x + 4 \equiv 4(2x + 1)$

Note:
$8x + 4 \equiv 2(4x + 2)$
This is factorised but the HCF has not been used

Solve equations with brackets

$3(2x + 4) = 30$



$3(2x + 4) = 30$

Expand the brackets

$6x + 12 = 30$
$-12 \qquad -12$

Substitute to check your answer. This could be negative or a fraction or decimal

$6x = 18$
$\div 6 \qquad \div 6$

$x = 3$

Simple Inequalities

$<$ less than
$\leq$ Less than or equal to
$>$ More than
$\geq$ More than or equal to

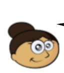
$x < 10$
Say this out loud
"x is a value less than 10"

$10 > x$
Say this out loud
"10 is more than the value"

Note:
$x < 10$ and $10 > x$ represent the same values

$x + 2 \leq 20$
"my value + 2 is less than or equal to 20"
$x \leq 18$
The biggest the value can be is 18

Form and solve inequalities



Two more than treble my number is greater than 11

Find the possible range of values

Form

x → x3 → +2 → 11

$3x + 2 > 11$

Solve

x ← ÷3 ← -2 ← 11

$x > 3$

Check

This would suggest any value bigger than 3 satisfies the statement

$3 \times 3 + 2 = 11$ ✓ $\qquad 10 \times 3 + 2 = 32$ ✓

Algebraic constructs

Expression
A sentence with a minimum of two numbers and one maths operation

Equation
A statement that two things are equal

Term
A single number or variable

Identity
An equation where both sides have variables that cause the same answer includes $\equiv$

Formula
A rule written with all mathematical symbols
eg area of a rectangle $A = b \times h$

YEAR 8 - ALGEBRAIC TECHNIQUES...

Sequences

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What do I need to be able to do?

By the end of this unit you should be able to:

- Generate a sequence from term to term or position to term rules
- Recognise arithmetic sequences and find the nth term
- Recognise geometric sequences and other sequences that arise

Keywords

Sequence: items or numbers put in a pre-decided order
Term: a single number or variable
Position: the place something is located
Linear: the difference between terms increases or decreases (+ or -) by a constant value each time
Non-linear: the difference between terms increases or decreases in different amounts, or by x or ÷
Difference: the gap between two terms
Arithmetic: a sequence where the difference between the terms is constant
Geometric: a sequence where each term is found by multiplying the previous one by a fixed non zero number

Linear and Non Linear Sequences

Linear Sequences – increase by addition or subtraction and the same amount each time

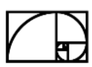
Non-linear Sequences – do not increase by a constant amount – quadratic, geometric and Fibonacci

- Do not plot as straight lines when modelled graphically
- The differences between terms can be found by addition, subtraction, multiplication or division

Fibonacci Sequence – look out for this type of sequence

0 1 1 2 3 5 8 ...

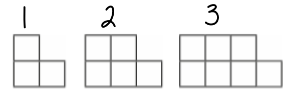
Each term is the sum of the previous two terms.



Sequence in a table and graphically

Position: the place in the sequence

1 2 3



3 5 7

"The term in position 3 has 7 squares"

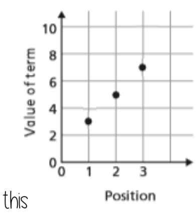
Term: the number or variable (the number of squares in each image)

In a table

Position	1	2	3
Term	3	5	7

+2 +2

Graphically



Because the terms increase by the same addition each time this is **linear** – as seen in the graph

Sequences from algebraic rules

This is substitution!

$3n + 7$

This will be linear - note the single power of n. The values increase at a constant rate

$3n^2 + 7$

This is not linear as there is a power for n

$2n - 5$ → Substitute the number of the term you are looking for in place of 'n'

eg
1st term = 2 (1) – 5 = -3
2nd term = 2 (2) – 5 = -1
100th term = 2 (100) – 5 = 195

Checking for a term in a sequence

Form an equation

Is 201 in the sequence 3n – 4?

$3n - 4 = 201$

Algebraic rule

Term to check

Solving this will find the position of the term in the sequence.
ONLY an integer solution can be in the sequence.

Complex algebraic rules

Misconceptions and comparisons

$2n^2$

2 times whatever n squared is

eg
1st term = 2 x 1² = 2
2st term = 2 x 2² = 8
100th term = 2 x 100² = 2000

$(2n)^2$

2 times n then square the answer

eg
1st term = (2 x 1)² = 4
2st term = (2 x 2)² = 16
100th term = (2 x 100)² = 40000

$n(n + 5)$ ←

eg
1st term = 1 (1 + 5) = 6
2st term = 2 (2 + 5) = 14
100th term = 100 (100 + 5) = 10500

You don't need to expand the expression

Finding the algebraic rule

This is the 4 times table → 4, 8, 12, 16, 20...

4n

7, 11, 15, 19, 22 ← This has the same constant difference – but is 3 more than the original sequence

$4n + 3$

$4n + 3$

This is the constant difference between the terms in the sequence

This is the comparison (difference) between the original and new sequence

YEAR 8 - ALGEBRAIC TECHNIQUES...

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Indices

What do I need to be able to do?

By the end of this unit you should be able to:

- Add/ Subtract expressions with indices
- Multiply expressions with indices
- Divide expressions with indices
- Know the addition law for indices
- Know the subtraction law for indices

Keywords

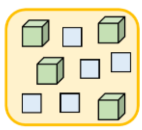
Base: The number that gets multiplied by a power
Power: The exponent — or the number that tells you how many times to use the number in multiplication
Exponent: The power — or the number that tells you how many times to use the number in multiplication
Indices: The power or the exponent.
Coefficient: The number used to multiply a variable
Simplify: To reduce a power to its lowest term
Product: Multiply

Addition/ Subtraction with indices

Coefficient → $5x^2 + 4x^4$ ← Power

Term, Term

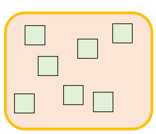
Expression



Each square represents x^2 and each cube represents x^4

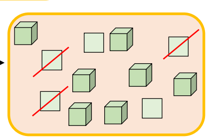
Only similar terms can be simplified
If they have different powers, they are unlike terms

$5x^2 + 2x^2$ →



→ $7x^2$

$5x^2 + 6x^4 - 3x^2 + x^4$ →



→ $2x^2 + 7x^4$

Multiply expressions with indices

$4b \times 3a$
$\equiv 4 \times b \times 3 \times a$
$\equiv 4 \times 3 \times b \times a$
$\equiv 12ab$

$5t \times 9t$
$\equiv 5 \times t \times 9 \times t$
$\equiv 5 \times 9 \times t \times t$
$\equiv 45t^2$

$2b^4 \times 3b^2$
$\equiv 2 \times b \times b \times b \times b \times 3 \times b \times b$
$\equiv 2 \times 3 \times b \times b \times b \times b \times b \times b$
$\equiv 6b^6$

There are often misconceptions with this calculation but break down the powers

Divide expressions with indices

$\frac{24}{36}$ → $\frac{2 \times 2 \times 2 \times 3}{2 \times 3 \times 2 \times 3}$ → $\frac{2}{3}$

$\frac{5a^3b^2}{15ab^6}$ → $\frac{5 \times a \times a \times a \times b \times b}{3 \times 5 \times a \times b \times b \times b \times b \times b \times b}$ → $\frac{a^2}{3b^4}$

Cross cancelling factors shows cancels the expression

$\frac{23a^7y^2}{5db^6}$

This expression cannot be divided (cancelled down) because there are no common factors or similar terms

Addition/ Subtraction laws for indices

$3^5 \times 3^2$ → 3^7
$= (3 \times 3 \times 3 \times 3 \times 3) \times (3 \times 3)$

The base number is all the same so the terms can be simplified

Addition law for indices

$$a^m \times a^n = a^{m+n}$$

$3^5 \div 3^2$ → 3^3

$\frac{3 \times 3 \times 3 \times 3 \times 3}{3 \times 3}$ → $\frac{3^3}{3^0}$ → $\frac{3^3}{1}$

Subtraction law for indices

$$a^m \div a^n = a^{m-n}$$

YEAR 8 - DEVELOPING NUMBER...

Fractions & Percentages

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Convert between FDP less than and more than 100.
- Increase or decrease using multipliers.
- Express an amount as a percentage.
- Find percentage change.

Keywords

Percent: parts per 100 – written using the % symbol

Decimal: a number in our base 10 number system. Numbers to the right of the decimal place are called decimals.

Fraction: a fraction represents how many parts of a whole value you have.

Equivalent: of equal value.

Reduce: to make smaller in value.

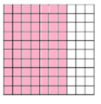
Growth: to increase/ to grow.

Integer: whole number, can be positive, negative or zero.

Invest: use money with the goal of it increasing in value over time (usually in a bank).

Convert FDP (R)

$\frac{70}{100}$ → This also means 70 ÷ 100 → 70 out of 100 squares → 70 "hundredths" = 7 "tenths" 0.7 →



→ 70 hundredths = 70%

Using a calculator

↓ ■/□ → S⇌D Convert to a decimal ↘ × 100 converts to a percentage

This will give you the answer in the simplest form

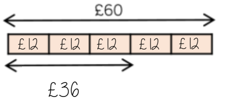
Be careful of recurring decimals

eg $\frac{1}{3}$ = 0.3333333 = $0.\dot{3}$

The dot above the 3

Fraction/ Percentage of amount (R)

Find $\frac{3}{5}$ of £60



Remember $\frac{3}{5}$ = 60%

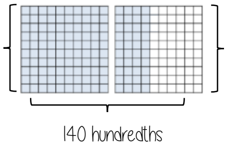
10% of £60 = £6
50% of £60 = £30
60% of £60 = £36

Remember $\frac{3}{5}$ = 60% = 0.6

60% of £60
= 0.6 x 60
= £36

Convert FDP < and > 100%

100 hundredths
10 tenths
100%

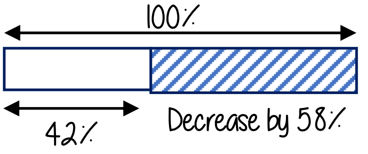


40 hundredths
4 tenths
40%

140 hundredths
14 tenths
140%

100%+40%
1 + 0.40
= 1.40

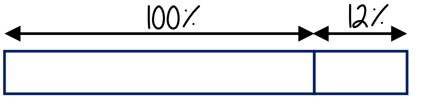
Percentage decrease: Multipliers



100% - 58% = 42%

1.00 – 0.58 = 0.42 ← Multiplier Less than 1

Percentage increase: Multipliers

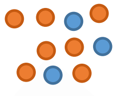


100% + 12% = 112%

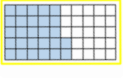
1.00 + 0.12 = 1.12 ← Multiplier More than 1

Express as a % - Non-calculator

Percent – per hundred



7 per every 10 are orange $\frac{7}{10}$ → This means that 70 per every 100 are orange $\frac{70}{100}$ → 70%



27 per every 50 shaded $\frac{27}{50}$ → 54 per every 100 shaded $\frac{54}{100}$ → 54%

Denominator 100 | Equivalent fractions

Express as a % - Calculator

Rosie

$\frac{13}{30}$ → ■/□ → $\frac{13}{30}$ → × 100 → 43.3333....% 43%

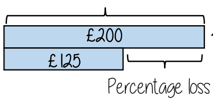
Can't use equivalence easily to find 'per hundred'

This the same as 13 ÷ 30

Decimal percentages are still a percentage

Percentage change

I bought a phone for £200. A year later sold it for £125



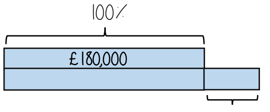
All values of change compare to the ORIGINAL value

Percentage loss

$\frac{75}{200}$ × 100 = 37.5%

$\frac{\text{Difference in value}}{\text{Original value}}$ × 100

I bought a house for £180,000, I later sold it for £216,000



Percentage profit

Money made (profit value) → $\frac{36000}{180000}$ × 100 = 20%

Choose appropriate method

The language and wording of the question is the key

Have you represented the question in a bar model?
Can you use a calculator?

YEAR 8 - DEVELOPING NUMBER...

Standard Form

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Write numbers in standard form and as ordinary numbers
- Order numbers in standard form
- Add/ Subtract with standard from
- Multiply/ Divide with standard form
- Use a calculator with standard form

Keywords

Standard (index) Form: A system of writing very big or very small numbers
Commutative: an operation is commutative if changing the order does not change the result.
Base: The number that gets multiplied by a power
Power: The exponent – or the number that tells you how many times to use the number in multiplication
Exponent: The power – or the number that tells you how many times to use the number in multiplication
Indices: The power or the exponent.
Negative: A value below zero.

Positive powers of 10

1 billion – 1 000 000 000
$10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^9$

Addition rule for indices $10^a \times 10^b = 10^{a+b}$

Subtraction rule for indices $10^a \div 10^b = 10^{a-b}$

Standard form with numbers > 1

Any number between 1 and less than 10 → $A \times 10^n$ ← Any integer

Example

3.2×10^4
$= 3.2 \times 10 \times 10 \times 10 \times 10$
$= 32000$

Non-example

0.8×10^4

$5.3 \times 10^{0.7}$

Negative powers of 10

0.001
$1 \times \frac{1}{1000}$
1×10^{-3}

10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
10^1	10^0	10^{-1}	10^{-2}	10^{-3}
0	0	0	0	1

Any value to the power 0 always = 1

Negative powers do not indicate negative solutions

Numbers between 0 and 1

0.054
$= 5.4 \times 10^{-2}$

1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
10^0	10^{-1}	10^{-2}	10^{-3}
0	0	5	4

A negative power does not mean a negative answer – it means a number closer to 0

Order numbers in standard form

10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}

6.4×10^{-2}	2.4×10^2	3.3×10^0	1.3×10^{-1}
0.064	240	1	0.13

Look at the power first will the number be = > or < than 1

Use a place value grid to compare the numbers for ordering

Mental calculations

$6.4 \times 10^2 \times 1000$ Not in Standard Form
$= 6.4 \times 10^2 \times 10^3$ Use addition for indices rule
$= 6.4 \times 10^5$

$8 \times 10^5 \times 3$
$= 24 \times 10^5$ Not in Standard Form
$= 2.4 \times 10^1 \times 10^5$ Use addition for indices rule
$= 2.4 \times 10^6$

$(2 \times 10^3) \div 4$ Divide the values
$= (2 \div 4) \times 10^3$
$= 0.5 \times 10^3$

Remember the layout for standard form

Any number between 1 and less than 10 → $A \times 10^n$ ← Any integer

Addition and Subtraction

Tip: Convert into ordinary numbers first and back to standard from at the end

$6 \times 10^5 + 8 \times 10^5$

Method 1

$= 600000 + 800000$
$= 1400000$
$= 1.4 \times 10^5$

More robust method
Less room for misconceptions
Easier to do calculations with negative indices
Can use for different powers

Method 2

$= (6 + 8) \times 10^5$
$= 14 \times 10^5$ ← This is not the final answer
$= 1.4 \times 10^1 \times 10^5$
$= 1.4 \times 10^5$

Only works if the powers are the same

Multiplication and division

$\frac{1.5 \times 10^5}{0.3 \times 10^3}$ ← Division questions can look like this

$(1.5 \times 10^5) \div (0.3 \times 10^3)$

$1.5 \div 0.3 \times 10^5 \div 10^3$

$= 5 \times 10^2$

For multiplication and division you can look at the values for A and the powers of 10 as two separate calculations

Revisit addition and subtraction laws for indices – they are needed for the calculations

Addition law for indices
$a^m \times a^n = a^{m+n}$

Subtraction law for indices
$a^m \div a^n = a^{m-n}$

Using a calculator

$1.4 \times 10^5 \times 3.9 \times 10^3$

Use a calculator to work out this question to a suitable degree of accuracy

Input 1.4 and press [x10ˣ] Then press 5 (for the power)
Press [×]
Input 3.9 and press [x10ˣ] Then press 3 (for the power)
Press [=]

This gives you the solution

Click calculator for video tutorial

To put into standard form and a suitable degree of accuracy

Press [SHIFT] [SETUP] and then press 7 for sci mode

Choose a degree of accuracy so in most cases press 2

Answer: 5.5×10^8

YEAR 8 – DEVELOPING NUMBER...

Number Sense

What do I need to be able to do?

By the end of this unit you should be able to:

- Round numbers to powers of 10 and 1 sf
- Round numbers to any dp
- Estimate solutions
- Calculate using order of operations
- Calculate with money, units of measurement and time

Keywords

Significant: Place value of importance
Round: Making a number simpler but keeping its value close to what it was.
Decimal: Place holders after the decimal point.
Overestimate: Rounding up – gives a solution higher than the actual value
Underestimate: Rounding down – gives a solution lower than the actual value.
Metric: A system of measurement.
Balance: The amount of money in a bank account
Deposit: Putting money into a bank account.

Round to powers of 10 and 1 sig figure

R If the number is halfway between we "**round up**"

5495 to the nearest 1000: 5000 ← ↑ — 6000 (answer 5000)

5475 to the nearest 100: 5400 — ↑ 5500 (answer 5500)

5475 to the nearest 10: 5470 — ↑ — 5480 (answer 5480)

370 to 1 significant figure is 400
37 to 1 significant figure is 40
3.7 to 1 significant figure is 4
0.37 to 1 significant figure is 0.4
0.00037 to 1 significant figure is 0.0004

Round to the first non-zero number

Round to decimal places

2.46192 ← Focus on the numbers **after** the decimal point

"To 1dp" – to one number after the decimal
"To 2dp" – to two numbers after the decimal

2.46192 (to 1dp) - Is this closer to 2.4 or 2.5

2.4 — ↑ — 2.5

2.4 | 6192 This shows the number is closer to 2.5

2.46192 (to 2dp) - Is this closer to 2.46 or 2.47

2.46 ↑ — 2.47

2.46 | 192 This shows the number is closer to 2.46

Estimate the calculation

Round to 1 significant figure to estimate

4.2 + 6.7 ≈ 4 + 7 ≈ 11 This is an **overestimate** because the 6.7 was rounded up more

The equal sign changes to show it is an estimation

21.4 x 3.1 ≈ 20 x 3 ≈ 60 This is an **underestimate** because both values were rounded down

It is good to check all calculations with an estimate in all aspects of maths – it helps you identify calculation errors.

Order of operations

R

Brackets Operations in brackets are calculated first

Other operations e.g. powers, roots,

Multiplication/ Division

They are carried out in the order from left to right in the question

Addition/ Subtraction

They are carried out in the order from left to right in the question

Calculations with money

Debit - You have £0 or more in an account

Credit - You have less than £0 in an account

Money calculations are to 2dp

Using a calculator – ensure you are working in the correct units.

£1.30 + 50p = 130 + 50 (in pence)
= 1.30 + 0.50 (in pounds)

£1 = 100p

Units are important:

Useful Conversions

mm → cm → m → km: ÷ 10, ÷ 100, ÷ 1000; km → m → cm → mm: × 1000, × 100, × 10

g → kg: ÷ 1000; kg → g: × 1000

ml → L: ÷ 1000; L → ml: × 1000

Metric measures of length

Kilo = 1000 x meter

Centi - $\frac{1}{100}$ x meter

Milli - $\frac{1}{1000}$ x meter

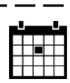
Units of weight/ capacity

Weight = g, kg, t
Capacity (volume of liquid) = ml, L

Time and the calendar

1 Year – the amount of time it takes Earth to go around the sun **365** (and a quarter) **days**

Leap Year – **366 days** (every 4 years)



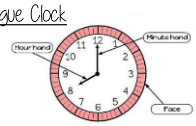
12 Months = one year = 52 weeks
31 days – **Jan, March, May, July Aug, Oct, Dec**
30 days – **April, June, Sept, Nov**
28 days – **Feb** (29 leap year)

1 week – 7 days
Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday

1 day – 24 hours
1 hour - 60 minutes
1 minute – 60 seconds

Use a number line for time calculations!

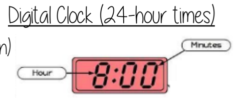
Analogue Clock



12-hour clock

- Use am (morning) and pm (afternoon)
- Only use hour times up to 12

Digital Clock (24-hour times)



24-hour clock

- 0-11 (morning hours)
- 12-23 (afternoon hours)

YEAR 8 – DEVELOPING GEOMETRY…

@whisto_maths

Angles in parallel lines and polygons

What do I need to be able to do?

By the end of this unit you should be able to:

- Identify alternate angles
- Identify corresponding angles
- Identify co-interior angles
- Find the sum of interior angles in polygons
- Find the sum of exterior angles in polygons
- Find interior angles in regular polygons

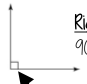
Keywords

Parallel: Straight lines that never meet
Angle: The figure formed by two straight lines meeting (measured in degrees)
Transversal: A line that cuts across two or more other (normally parallel) lines
Isosceles: Two equal size lines and equal size angles (in a triangle or trapezium)
Polygon: A 2D shape made with straight lines
Sum: Addition (total of all the interior angles added together)
Regular polygon: All the sides have equal length; all the interior angles have equal size.

Basic angle rules and notation

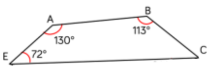
R

Acute Angles
0°< angle <90°



Right Angles
90°

Right angle notation



The letter in the middle is the angle
The arc represents the part of the angle

Angle Notation: three letters **ABC**
This is the angle at B = 113 °
Line Notation: two letters **EC**
The line that joins E to C.

Obtuse
90°< angle <180°

Reflex
180°< angle <360°

Straight Line
180°

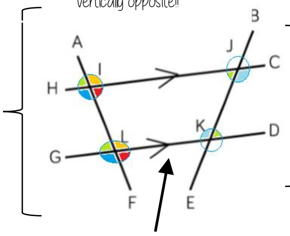
Vertically opposite angles
Equal
Angles around a point
360 °

Parallel lines

Still remember to look for angles on straight lines, around a point and vertically opposite!!

Lines AF and BE are **transversals** (lines that bisect the parallel lines)

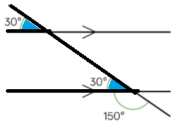
Corresponding angles often identified by their **"F shape"** in position



Alternate angles often identified by their **"Z shape"** in position

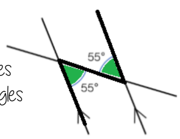
This notation identifies parallel lines

Alternate/ Corresponding angles

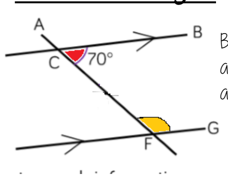


Because alternate angles are equal the highlighted angles are the same size

Because corresponding angles are equal the highlighted angles are the same size



Co-interior angles



Because co-interior angles have a sum of 180° the highlighted angle is 110°

As angles on a line add up to 180° co-interior angles can also be calculated from applying alternate/ corresponding rules first

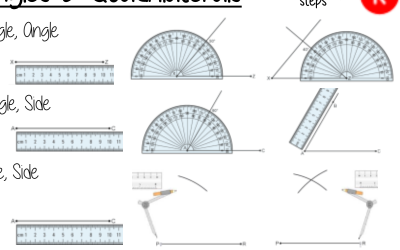
Triangles & Quadrilaterals

Link to steps R

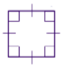
Side, Angle, Angle

Side, Angle, Side

Side, Side, Side



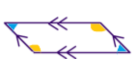
Properties of Quadrilaterals



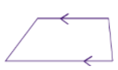
Square
All sides equal size
All angles 90°
Opposite sides are parallel

Rectangle
All angles 90°
Opposite sides are parallel

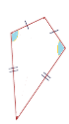
Rhombus
All sides equal size
Opposite angles are equal



Parallelogram
Opposite sides are parallel
Opposite angles are equal
Co-interior angles



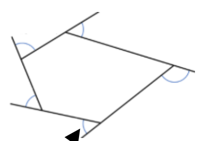
Trapezium
One pair of parallel lines



Kite
No parallel lines
Equal lengths on top sides
Equal lengths on bottom sides
One pair of equal angles

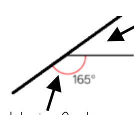
Sum of exterior angles

Exterior angles all add up to 360°



Exterior Angles
Are the angle formed from the straight-line extension at the side of the shape

Using exterior angles



Exterior Angle

Interior Angle

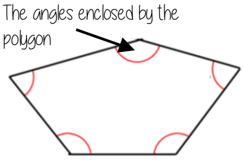
Interior angle + Exterior angle = straight line = 180°
Exterior angle = 180 – 165 = 15°

Number of sides = 360° ÷ exterior angle
Number of sides = 360 ÷ 15 = 24 sides

Sum of interior angles

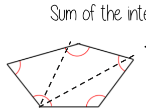
(number of sides – 2) x 180

Interior Angles
The angles enclosed by the polygon



This is an **irregular** polygon – the sides and angles are different sizes

Sum of the interior angles = (5 – 2) x 180

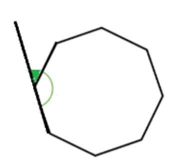


This shape can be made from three triangles
Each triangle has 180°

Sum of the interior angles = 3 x 180
= 540°

Remember this is **all** of the interior angles added together

Missing angles in regular polygons



Exterior angle = 360 ÷ 8 = 45°

Interior angle = $\frac{(8-2) \times 180}{8} = \frac{6 \times 180}{8} = 135°$

Exterior angles in regular polygons = 360° ÷ number of sides

Interior angles in regular polygons = $\frac{(\text{number of sides} - 2) \times 180}{\text{number of sides}}$

YEAR 8 - DEVELOPING GEOMETRY...

Area of trapezia and Circles

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Recall area of basic 2D shapes
- Find the area of a trapezium
- Find the area of a circle
- Find the area of compound shapes
- Find the perimeter of compound shapes

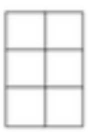
Keywords

Congruent: The same
Area: Space inside a 2D object
Perimeter: Length around the outside of a 2D object
Pi (π): The ratio of a circle's circumference to its diameter.
Perpendicular: At an angle of 90° to a given surface
Formula: A mathematical relationship/ rule given in symbols. E.g. b x h = area of rectangle/ square
Infinity (∞): A number without a given ending (too great to count to the end of the number) — never ends
Sector: A part of the circle enclosed by two radii and an arc.

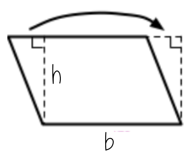
Area – rectangles, triangles, parallelograms

R

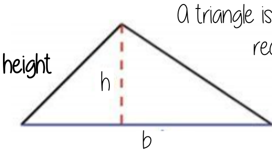
Rectangle
Base x Height



Parallelogram/ Rhombus
Base x Perpendicular height



Triangle
½ x Base x Perpendicular height

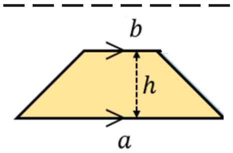


A triangle is half the size of the rectangle it would fit in

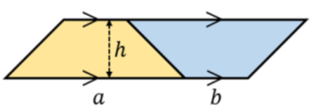
Area of a trapezium

Area of a trapezium

$$\frac{(a+b) \times h}{2}$$



Why?

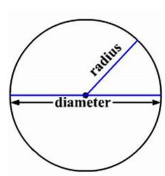


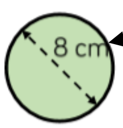
- Two congruent trapeziums make a parallelogram
- New length (a + b) x height
- Divide by 2 to find area of one

Area of a circle (Non-Calculator)

Read the question – leave in terms of π or if $\pi \approx 3$ (provides an estimate for answers)

Area of a circle
π x radius²



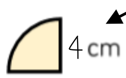


Diameter = 8cm
∴ **Radius** = 4cm

π x radius²
= π x 4²
= π x 16
= 16π cm²

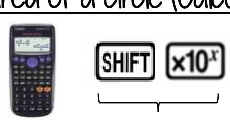
Find the area of one quarter of the circle

Radius = 4cm



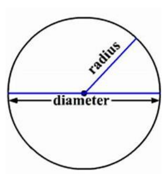
Circle Area = 16π cm²
Quarter= 4π cm²

Area of a circle (Calculator)



How to get π symbol on the calculator

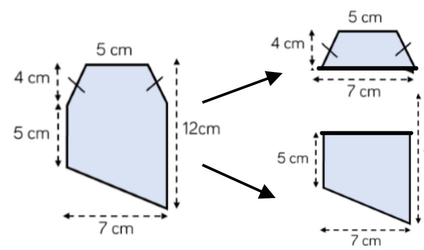
Area of a circle
π x radius²



It is important to round your answer suitably – to significant figures or decimal places. This will give you a decimal solution that will go on forever!

Compound shapes

To find the area compound shapes often need splitting into more manageable shapes first. Identify the shapes and missing sides etc. first.



Shape A - Isosceles trapezium

Shape B- nonstandard trapezium

Units

Shape A + Shape B = total area

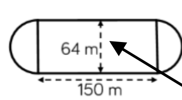
$\frac{(5 + 7) \times 4}{2}$ + $\frac{(5 + 8) \times 7}{2}$ = 24 + 45.5 = 69.5cm²

Compound shapes including circles

Circumference
π x diameter

Compound shapes are not always area questions. For Perimeter you will need to use the circumference

Spotting diameters and radii



This dimension is also the diameter of the semi circles

Arc lengths = π x 64
= 64 π

Don't need to halve this because there are 2 ends which make the whole circle

Arc lengths + Straight lengths = total perimeter

= 64 π + 150 + 150
= (300 + 64 π) m
OR = 501.1 m

Still remember to split up the compound shape into smaller more manageable individual shapes first

YEAR 8 - DEVELOPING GEOMETRY...

Line symmetry and reflection

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

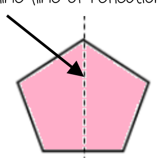
- Recognise line symmetry
- Reflect in a horizontal line
- Reflect in a vertical line
- Reflect in a diagonal line

Keywords

Mirror line: a line that passes through the center of a shape with a mirror image on either side of the line
Line of symmetry: same definition as the mirror line
Reflect: mapping of one object from one position to another of equal distance from a given line.
Vertex: a point where two or more-line segments meet.
Perpendicular: lines that cross at 90°
Horizontal: a straight line from left to right (parallel to the x axis)
Vertical: a straight line from top to bottom (parallel to the y axis)

Lines of symmetry

Mirror line (line of reflection)

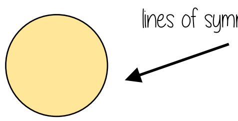


Rhombus
two lines of symmetry

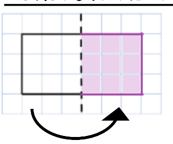
Parallelogram
No lines of symmetry

Shapes can have more than one line of symmetry.... This regular polygon (a regular pentagon has 5 lines of symmetry)

A circle has an infinite amount of lines of symmetry

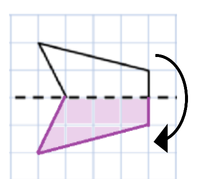


Reflect horizontally/ vertically (1)



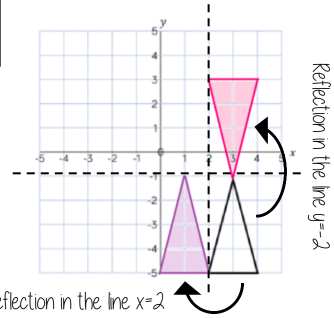
Note: a reflection doubles the area of the original shape

Reflection in a vertical line



Reflection in a horizontal line

Reflection on an axis grid

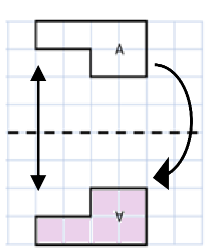


Reflection in the line y=-2

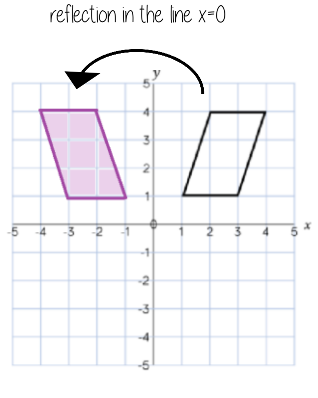
Reflection in the line x=2

Reflect horizontally/ vertically (2)

All points need to be the same distance away from the line of reflection



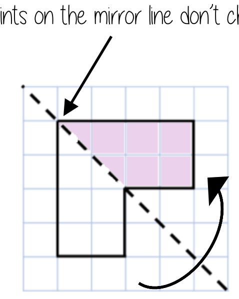
Reflection in the line y axis – this is also a reflection in the line x=0



Lines parallel to the x and y axis
REMEMBER
Lines parallel to the x-axis are y = ____
Lines parallel to the y-axis are x = ____

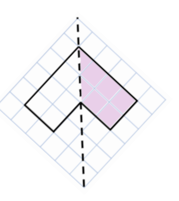
Reflect Diagonally (1)

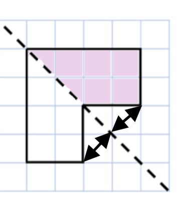
Points on the mirror line don't change position



Fold along the line of symmetry to check the direction of the reflection

Turn your image
If you turn your image it becomes a vertical/ horizontal reflection (also good to check your answer this way)

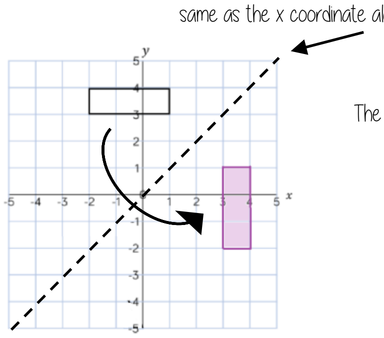




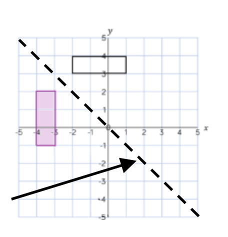
Drawing perpendicular lines
Perpendicular lines to and from the mirror line can help you to plot diagonal reflections

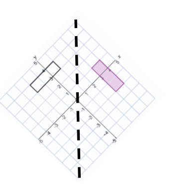
Reflect Diagonally (2)

This is the line **y = x** (every y coordinate is the same as the x coordinate along this line)



This is the line **y = - x**
The x and y coordinate have the same value but opposite sign





Turn your image
If you turn your image it becomes a vertical/ horizontal reflection (also good to check your answer this way)

YEAR 8 - REASONING WITH DATA...

The data handling cycle

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Set up a statistical enquiry
- Design and criticise questionnaires
- Draw and interpret multiple bar charts
- Draw and interpret line graphs
- Represent and interpret grouped quantitative data
- Find and interpret the range
- Compare distributions

Keywords

Hypothesis: an idea or question you want to test
Sampling: the group of things you want to use to check your hypothesis
Primary Data: data you collect yourself
Secondary Data: data you source from elsewhere e.g the internet/ newspapers/ local statistics
Discrete Data: numerical data that can only take set values
Continuous Data: numerical data that has an infinite number of values (often seen with height, distance, time)
Spread: the distance/ how spread out/ variation of data
Average: a measure of central tendency – or the typical value of all the data together
Proportion: numerical relationship that compares two things

Set up a statistical enquiry

Write a suitable hypothesis → Design a data collection sheet → Pros/ Cons of sampling → Pros/ Cons primary or secondary data → Discrete or continuous data?

Features of a data collection sheet

Data Title	Tally	Frequency
	~~IIII~~	

Grouped or ungrouped categories

Total number of that group observed

Design and criticise a questionnaire

The Question - be clear with the question - don't be too leading/ judgemental

e.g How much pocket money do you get a week?

Responses – do you want closed or open responses? – do any options overlap? – Have you an option for all responses?

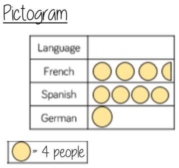
Zero option → ☐ £0 ☐ £0.01 - £2 ☐ £2.01 - £4 ☐ more than £4 ← More option

NOTE: For responses about continuous data include inequalities $< x \leq$

Pictograms, bar and line charts

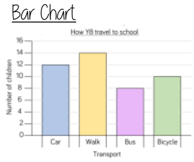
Represents quantitative data

Pictogram



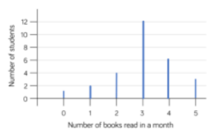
- Need to remember a key
- Visually able to identify mode

Bar Chart



- Gaps between the bars
- Clearly labelled axes
- Scale for the axes
- Title for the bar chart
- Discrete Data

Line Chart

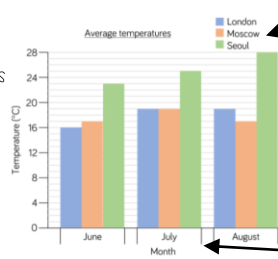


- Gaps between the lines
- Clearly labelled axes
- Scale for the axes
- Discrete Data

Multiple Bar chart

Compares multiple groups of data

- Clearly labelled axes
- Scale for axes
- Comparable data bars drawn next to each other



Key/ Colour code for separate groups of information

Gap between different categories of data

Draw and interpret Pie Charts

Remember a circle has 360°

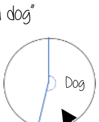
Type of pet	Dog	Cat	Hamster
Frequency	32	25	3

There were 60 people asked in this survey (Total frequency)

$\frac{32}{60}$ "32 out of 60 people had a dog"

This fraction of the 360 degrees represents dogs

$\frac{32}{60} \times 360 = 192°$



Use a protractor to draw
This is 192°

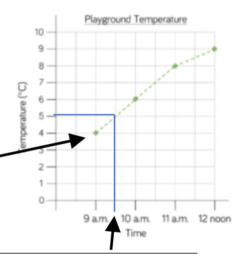
Multiple method
As 60 goes into 360 – 6 times.
Each frequency can be multiplied by 6 to find the degrees (proportion of 360)

Represents quantitative, discrete data

Draw and interpret line graphs

- Commonly used to show changing over time
- The points are the recorded information and the lines join the points.

Line graphs do not need to start from 0



More than one piece of data can be plotted on the same graph to compare data

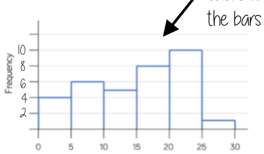
It is possible to make estimates from the line e.g temperature at 9.30am is 5°C

Grouped quantitative data

Time (minutes)	Frequency
$0 \leq t < 5$	4
$5 \leq t < 10$	6
$10 \leq t < 15$	5
$15 \leq t < 20$	8
$20 \leq t < 25$	10
$25 \leq t < 30$	1

"More than or equal to 25 and less than 30 minutes"

The use of inequalities shows that this will be a frequency diagram



This is a frequency diagram
There are no gaps between the bars

Grouping the data is useful if there is a large spread of data to begin with

Find and interpret the range

Difference between the biggest and smallest values

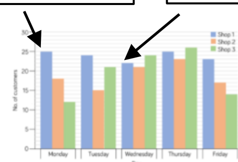
The range is a measure of **spread**

A smaller range means there is less variation in the results – it is more consistent data

A range of 0 means all the data is the same value

Shop 1 highest value

Shop 1 lowest value



Shop 1 has the smallest range – this indicates it has a more consistent flow of customers each week

Range of customers = 25 – 22 = 3 (Shop 1)

YEAR 8 - REASONING WITH DATA...

Measures of location

@whisto_maths

What do I need to do?

By the end of this unit you should be able to:

- Understand and use mean, median and mode
- Choose the most appropriate average
- Identify outliers
- Compare distributions using averages and range

Keywords

Spread: the distance/ how spread out/ variation of data
Average: a measure of central tendency – or the typical value of all the data together
Total: all the data added together
Frequency: the number of times the data values occur
Represent: something that show's the value of another
Outlier: a value that stands apart from the data set
Consistent: a set of data that is similar and doesn't change very much

Mean, Median, Mode

The Mean

A measure of average to find the central tendency... a typical value that represents the data

24, 8, 4, 11, 8,

Find the sum of the data (add the values) 55

Divide the overall total by how many pieces of data you have 55 ÷ 5

Mean = 11

The Median

The value in the center (in the middle) of the data

24, 8, 4, 11, 8,

Put the data in order 4, 8, 8, 11, 24

Find the value in the middle 4, 8, (8), 11, 24

Median = 8

NOTE: If there is no single middle value find the mean of the two numbers left

The Mode (The modal value)

This is the number OR the item that occurs the most (it does not have to be numerical)

24, 8, 4, 11, 8,

This can still be easier if it the data is ordered first

4, 8, 8, 11, 24

Mode = 8

Choosing the appropriate average

The average should be a representative of the data set – so it should be compared to the set as a whole - to check if it is an appropriate average

Here are the weekly wages of a small firm

£240	£240	£240	£240	£240
£260	£260	£300	£350	£700

Which average best represents the weekly wage?

The Mean = £307
The Median = £250
The Mode = £240

Put the data back into context
Mean/Median – too high (most of this company earn £240)
Mode is the best average that represents this wage

It is likely that the salaries above £240 are more senior staff members – their salary doesn't represent the average weekly wage of the majority of employers

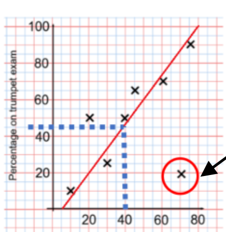
Identify outliers

Outliers are values that stand well apart from the rest of the data

Outliers can have a big impact on range and mean. They have less impact on the median and the mode ← Sometimes it is best to not use an outlier in calculations

Height in cm
152 150 142 158 (182) 151 153 149 156 160 151 144

Where an outlier is identified try to give it some context.
This is likely to be a taller member of the group.
Could the be an older student or a teacher?



Outliers can also be identified graphically eg on scatter graphs

Comparing distributions

Comparisons should include a statement of average and central tendency, as well as a statement about spread and consistency.

Here are the number of runs scored last month by Lucy and James in cricket matches
Lucy: 45, 32, 37, 41, 48, 35
James: 60, 90, 41, 23, 14, 23

Lucy
Mean: 39.6 (1dp), **Median**: 38 **Mode**: no mode, **Range**: 16

James
Mean: 41.8 (1dp), **Median**: 32, **Mode**: 23, **Range**: 76 ← James has two extreme values that have a big impact on the range

"James is less consistent that Lucy because his scores have a greater range. Lucy performed better on average because her scores have a similar mean and a higher median"